WS – Solids of Revolution

1. Let R be the region in the first quadrant that is enclosed by the graphs of $y = 4 - x^2$, $y = 3x$ and the y-axis.
   a. Find the exact area of R.
   b. Find the volume of the solid obtained by revolving R about the x-axis.
   c. Find the volume of the solid obtained by revolving R about the y-axis.
   d. Find the volume of the solid obtained by revolving R about the line $y = -2$.
   e. Find the volume of the solid obtained by revolving R about the line x = 5

2. Let R be the region enclosed by that is enclosed by the graphs of $f(x) = 2x - \frac{x^2}{2}, g(x) = e^{x-2}$.
   a. Find the area R.
   b. Find the volume when R is revolved over the x-axis.
   c. Find the volume when R is revolved over the line y = - 2.
   d. Find the volume when R is revolved over the line y = 5.

3. Let R be the region in the first quadrant that is enclosed by the graphs of $y = 4 - x^2$, $y = 3x$ and the x-axis.
   a. Find the exact area of R.
   b. Find the volume of the solid obtained by revolving R about the x-axis.
   c. Find the volume of the solid obtained by revolving R about the y-axis.
   d. Find the volume of the solid obtained by revolving R about the line $x = 2$.
   e. Find the volume of the solid obtained by revolving R about the line y = 6.

4. Let R be the region in the first quadrant that is enclosed by the graphs of $y = \sqrt{x}$, $y = 4 - 3x^2$ and the y-axis.
   a. Find the exact area of R.
   b. Find the volume of the solid obtained by revolving R about the x-axis.
   c. Find the volume of the solid obtained by revolving R about the y-axis.
   d. Find the volume of the solid obtained by revolving R about the line $y = 5$.
   e. Find the volume of the solid obtained by revolving R about the line x = 4.

5. Let R be the region in the first quadrant that is enclosed by the graphs of $y = x + 1$, $y = 2x^3$ and the y-axis. Set up the integral, but do not evaluate.
   a. Find the exact area of R.
   b. Find the volume of the solid obtained by revolving R about the x-axis.
   c. Find the volume of the solid obtained by revolving R about the y-axis.
   d. Find the volume of the solid obtained by revolving R about the line $y = 5$.
   e. Find the volume of the solid obtained by revolving R about the line x = -2.

Answers

1. –
   a. 13/6
   b. 158pi/15
   c. 3pi/2
   d. 96pi/5
   e. 20.167pi

2. –
   a. $\int_a^b f(x)-g(x)dx = 2.149$
   b. $\pi\int_a^b f(x)^2 - g(x)^2\,dx = 4.4877\pi$
   c. $\pi\int_a^b \left(2+f(x)\right)^2 - \left(2+g(x)\right)^2\,dx = 13.472\pi$
   d. $\pi\int_a^b \left(5-g(x)\right)^2 - \left(5-f(x)\right)^2\,dx = 16.612\pi$

3. -
   a. 19/6
   b. 98pi/15
   c. 13pi/2
   d. 37pi/6
   e. 472pi/15

4. –
   a. 7/3
   b. 93pi/10
   c. 17pi/10
   d. $\approx 14.03\pi$
   e. 16.967pi

5. –
   a. $\int_0^1 \left[\left(x+1\right) - 2x^3\right]\,dx$
   b. $\pi\int_0^1 \left[\left(x+1\right)^2 - \left(2x^3\right)^2\right]\,dx$
   c. $\pi\int_0^1 \left(\frac{y}{2}\right)^{\frac{2}{3}} dy + \pi\int_1^2 \left[\left(\frac{y}{2}\right)^{\frac{2}{3}} - \left(y-1\right)^2\right]\,dy$
   d. $\pi\int_0^1 \left(5-2x^3\right)^2 - \left(4-x\right)^2\,dx$
   e. $\pi\int_0^1 \left(2+\sqrt[3]{y/2}\right)^2 - 2^2\,dy + \pi\int_1^2 \left(2+\sqrt[3]{y/2}\right)^2 - \left(2+\left(y-1\right)\right)^2\,dy$