Integration by Substitution – Show your work on your own paper.

Evaluate each definite integral. Work without a calculator. Then, check your answer with your calculator.

1. $\int_0^{\frac{\pi}{3}} \frac{\sin\theta}{\cos^2\theta}\,d\theta$
2. $\int_0^{\pi} \sin\frac{x}{3}\,dx$
3. $\int_0^{1} x\sqrt{1-x^2}\,dx$
4. $\int_0^{-4} \sqrt{1-2x}\,dx$
5. $\int_1^{2} (x-1)\sqrt{2-x}\,dx$
6. $\int_0^{4} \frac{1}{\sqrt{2x+1}}dx$
7. $\int_0^{2} \frac{x}{\sqrt{1+2x^2}}dx$

Integrate each.

8. $\int \cos\pi x\,dx$
9. $\int x\sqrt{x+3}\,dx$
10. $\int \frac{2-x}{\sqrt[3]{x}}dx$
11. $\int x\sqrt[3]{1-x}\,dx$
12. $\int \sec x\tan x\sqrt{1+\sec x}\,dx$
13. $\int \cos^4 x\,\sin x\,dx$
14. $\int \sin(2x+3)\,dx$
15. $\int \frac{(1+\sqrt{x})^4}{\sqrt{x}}dx$

Solve the differential equation.

16. $\frac{dy}{dx} = \sin 2x$
17. $\frac{dy}{dx} = \frac{10}{(2x-1)^2}$

18. Find f(x) given $f'(x) = \frac{6}{\sqrt{3x-2}}$ such that f(x) passes through (2,5).

Recall

19. Given the function defined by $g(x) = 3x^5 - 10x^4 + 7$, use derivatives to find
    a. The intervals that g is increasing
    b. The x-coordinates of any inflection points.

Answers

1. 1
2. 3/2
3. 1/3
4. -26/3
5. 4/15
6. 2
7. 1
8. $\frac{1}{\pi}\sin\pi x + C$
9. $\frac{2}{5}(x-2)(x+3)^{3/2} + C$
10. $3x^{2/3} - \frac{3}{5}x^{5/3} + C$

    Note: no substitution required!
11. $\frac{-3}{28}(1-x)^{4/3}(4x+3) + C$
12. $\frac{2}{3}(1+\sec x)^{3/2} + C$
13. $-\frac{1}{5}\cos^5 x + C$
14. $-\frac{1}{2}\cos(2x+3) + C$
15. $\frac{2}{5}(\sqrt{x}+1)^5 + C$
16. $y = -\frac{1}{2}\cos 2x + C$
17. $y = \frac{-5}{2x-1} + C$
18. $f(x) = 4\sqrt{3x-2} - 3$
19. –

    a. $(-\infty, 0)\left(\frac{8}{3}, \infty\right)$

    b. $x = 2$ only