**WS – Riemann Sums and the Definite Integral**
***AP Calculus - Harter***

Name_____
Date _____Per_____

1. Sketch a picture of $\int_{-1}^{4} x^2 + 1 \; dx$

2. If $\int_{0}^{5} f(x)\,dx = 10$ and $\int_{5}^{7} f(x)\,dx = 3$, find each

   a. $\int_{0}^{7} f(x)\,dx =$

   b. $\int_{5}^{0} f(x)\,dx =$

   c. $\int_{7}^{0} 2f(x)\,dx =$

   d. $\int_{5}^{5} f(x)\,dx =$

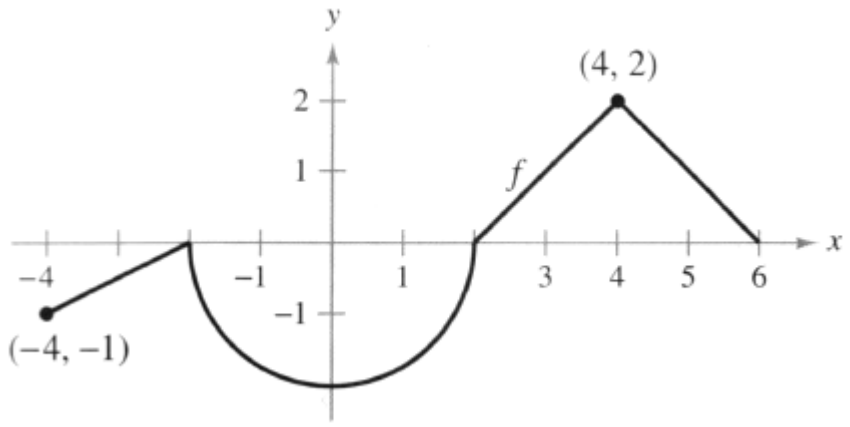
3. If $\int_{-1}^{1} f(x)\,dx = 1$ and $\int_{0}^{1} f(x)\,dx = 5$, find each

   a. $\int_{-1}^{0} f(x)\,dx =$

   b. $\int_{1}^{0} f(x)\,dx - \int_{-1}^{0} f(x)\,dx$

   c. $\int_{-1}^{1} 3f(x)\,dx =$

   d. $\int_{-1}^{0} f(x) + 2 \; dx =$



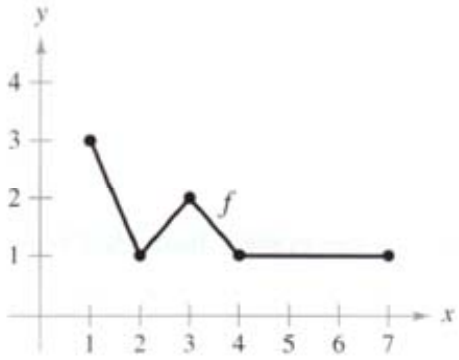

4. Use the picture above to evaluate each integral.

   a. $\int_{0}^{2} f(x)\,dx =$

   b. $\int_{-4}^{2} f(x)\,dx =$

   c. $\int_{-4}^{6} f(x)\,dx =$

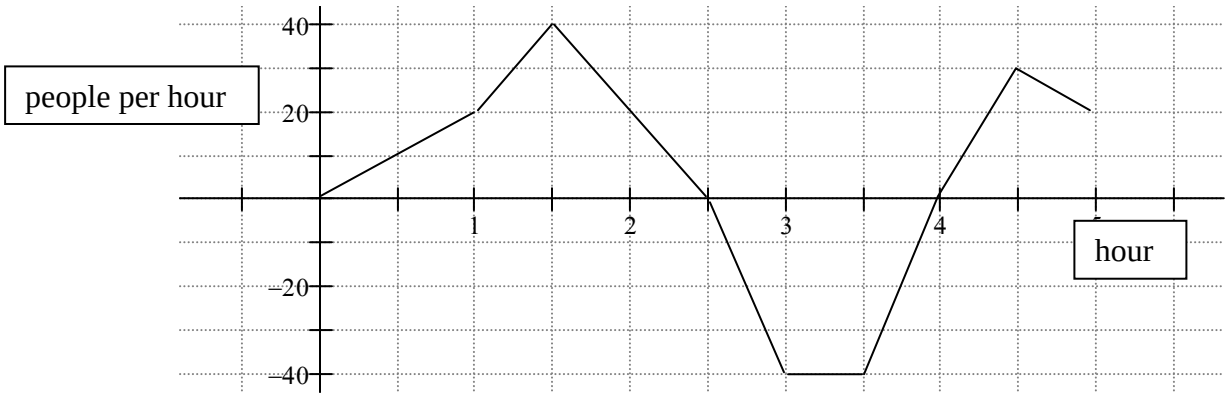
   d. $\int_{-4}^{6} f(x) + 2 \; dx =$




5. Use f at the left to find each.

   a. $\int_{1}^{7} f(x)\,dx =$

   b. The average value of f(x) on [1,7]




6. The rate that people enter a theater starting at noon is recorded above. There are 60 people in the theater at noon.
   a. How many were in the theatre at 2:00?
   b. How many people were in the theatre at 4:30?
   c. At what time were the most people in the theater? How many?

7. The table at the right gives the values for the rate (in gal/s) which water flowed into a lake with readings taken at specific times.

| Time (sec) | 0 | 5 | 12 | 20 | 36 | 42 |
|---|---|---|---|---|---|---|
| Rate (gal/sec) | 300 | 200 | 150 | 100 | 150 | 250 |

   a. Use a ***Left Hand Riemann Sum*** with 5 subintervals to estimate the total amount of water that flowed into the lake during the time period t = 0 to t = 42.

   b. Use a ***Trapezoidal Approximation*** with 5 subintervals to estimate the total amount of water that flowed into the lake during the time period t = 0 to t = 42.

8. Evaluate:
   a. $\int_{-1}^{2} 6x^2 + 2x - 1 \; dx$
   b. $\int_{1}^{8} \frac{2}{\sqrt[3]{x}} \; dx$
   c. $\int_{0}^{\frac{\pi}{2}} x - \sin x \; dx$

9. The table gives the values for the rate (in m/s) at which a car travels at specific times. Use **a Midpoint Riemann Sum** with 3 subintervals to estimate the distance the car traveled from t = 0 to t = 120.

| Time (sec) | 0 | 20 | 40 | 60 | 80 | 100 | 120 |
|---|---|---|---|---|---|---|---|
| Rate (gal/s) | 300 | 200 | 150 | 100 | 150 | 250 | 110 |

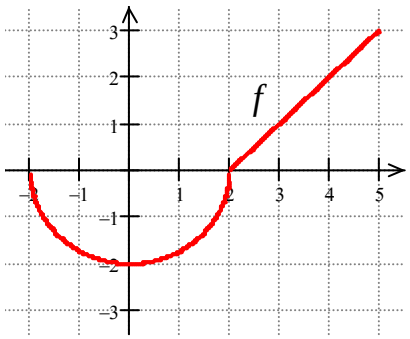
10. Find the **average value** of f on the given interval.

a. $f(x)=2x^2-3x$ on the interval [-2,1].

b. $f(x)=2x-\frac{2}{x^2}$ on the interval [1,4].

11. Solve the differential equation if $f''(x)=3x^2-3$, $f(2)=4$ and $f'(-1)=3$.

12. $g(x)=\int_0^x f(t)\,dt$ and f is graphed at the right. Find each.




a. $g(0)$

b. $g(2)$

c. $g(-2)$

d. $g(5)$

e. $g'(4)$

f. Find the equation of the line tangent to g(x) when x = 5.

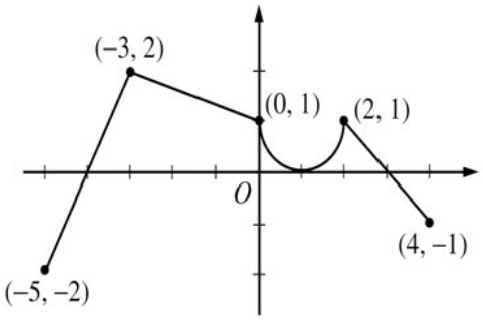
g. $g''(2)$

h. $g''(4)$

i. Where is g concave down?

j. Where is g increasing?

13. The graph of the function $f$ shown above consists of a semicircle and three line segments. Let $g$ be the function given by $g(x)=\int_{-3}^{x} f(t)\,dt$.



Graph of $f$

(a) Find $g(0)$ and $g'(0)$.

(b) Find all values of $x$ in the open interval $(-5, 4)$ at which $g$ attains a relative maximum. Justify your answer.

(c) Find the absolute minimum value of $g$ on the closed interval $[-5, 4]$. Justify your answer.

(d) Find all values of $x$ in the open interval $(-5, 4)$ at which the graph of $g$ has a point of inflection.