1st/2nd Derivative Test, Rolles Thm, MVT
SHOW YOUR WORK ON YOUR OWN PAPER

Examine the values of a continuous and differentiable function f shown below on the interval $[0,8]$.

x	0	1	2	3	4	5	6	7	8
$f(x)$	2	3	3	4	5	6	7	5	11

1. At how many values of x must there be a slope of zero on f(x)? Explain your reasoning.
2. On the interval $[0,8]$, does there have to exist a value of x where $f'(x)=\frac{9}{8}$? Explain your reasoning.

Use the first derivative test to name the largest open intervals where the function is decreasing or increasing. Name the x values all relative extrema.

3. $g(x)=2x^3-3x^2-36x-2$
4. $g(x)=\dfrac{x+2}{\sqrt{x}}$
5. $f(x)=3x^4-4x^3-12x^2+5$

Find the open intervals where the function is concave up or concave down. Name any points of inflection.

6. $f(x)=\frac{1}{5}x^5-2x^4$
7. $f(x)=-3x^5+15x^3$
8. $f(x)=x-2\sin x$; $[0,3\pi]$
9. $f(x)=\dfrac{x+2}{\sqrt{x}}$

Find the absolute extrema on the given interval.

10. $f(x)=\sin x+\cos x$; $\left[0,\frac{\pi}{3}\right]$
11. $f(x)=x^2+\frac{2}{x}$; $[1/2,2]$
12. Find all values of c where the **Mean Value Theorem** applies to the equation $f(x)=x^2-6x+1$ on the interval [-1, 2]. Draw a picture of f, the secant line and the tangent line that demonstrate your answer.
13. Find all values of c where the **Mean Value Theorem** applies to the equation $f(x)=\dfrac{x^2-1}{x+2}$ on the interval [-3, 2].
14. Find all values of c where the **Mean Value Theorem** applies to the equation $f(x)=x-\sin 2x$ on the interval [0, π/2].

Perform the **second** derivative test for each.

15. $f(x)=3x^4-8x^3$
16. $f(x)=x^4-8x^2+10$
17. Draw your own picture and use it to explain the Mean Value Theorem in your own words.
18. Draw a picture and and use it to explain Rolles Theorem in your own words.

Determine if Rolle's Theorem can be applied on the closed interval and find the value of c guaranteed by the Theorem for

19. $f(x)=\sin x$ on $[0,2\pi]$.
20. $f(x)=x^3-3x^2+2x+2$ on $[0,1]$.

ANSWERS

1. –
2. –
3. Rel MAX at x=-2, rel MIN at x=3
   INC $(-\infty,-2)$ $(3,\infty)$, DEC $(-2,3)$
4. Rel MIN at x=2
   INC $(2,\infty)$ DEC $(0,2)$
5. Rel MAX at x=0, rel MIN at x=-1,2
   DEC $(-\infty,-1)$ $(0,2)$ INC $(-1,0)$ $(2,\infty)$
6. POI at x = 6
   CCUP $(6,\infty)$ CCDWN $(-\infty,6)$
7. POI at $\pm\frac{\sqrt{6}}{2},0$
   CCUP $\left(-\infty,\frac{-\sqrt{6}}{2}\right),\left(0,\frac{\sqrt{6}}{2}\right)$ CCDWN $\left(\frac{-\sqrt{6}}{2},0\right),\left(\frac{\sqrt{6}}{2},\infty\right)$
8. POI at π, 2π
   CCUP $(0,\pi)$ $(2\pi,3\pi)$ CCDWN $(\pi,2\pi)$
9. POI AT X = 6
   CCUP $(0,6)$ CCDWN $(6,\infty)$
10. $f\left(\frac{\pi}{4}\right)=\sqrt{2}$; $f(0)=1$
11. $f(2)=5$; $f(1)=3$
12. x = ½
13. Rolles Thm does not apply since it is not continuous at -2
14. π/4
15. $f''(0)=0$ TEST FAILS!
    $f''(2)$ is POSITIVE, REL MIN at x=2
16. $f''(0)$ is neg, rel MAX at x=0
    $f''(-2)$ is pos, rel MIN at x=-2
    $f''(2)$ is pos, rel MIN at x=2
17. See your notes
18. See your notes
19. f(x) is continuous and differentiable on $[0,2\pi]$ and $f(0)=f(2\pi)=0$, so Rolles THM applies.
    There is zero slope at $\frac{\pi}{2}$ and $\frac{3\pi}{2}$ on $[0,2\pi]$.
20. $1-\frac{\sqrt{3}}{3}$