Second Derivative Test and Concavity

Find the points of inflection and discuss the concavity of each.

1. $f(x)=x^3-6x^2+12x$
2. $f(x)=2x^3-3x^2-12x+5$
3. $f(x)=\frac{1}{4}x^4-2x^2$
4. $f(x)=2x^4-8x+3$
5. $f(x)=\frac{x}{x^2+1}$ (video)
6. $f(x)=\frac{x+1}{\sqrt{x}}$
7. $f(x)=\sin x+\cos x\ ;\ [0,2\pi]$

Use the 2nd derivative test to find all relative extrema.

8. $f(x)=x^4-4x^3+2$
9. $f(x)=x^2+3x+8$
10. $f(x)=(x-5)^2$
11. $f(x)=x^3-3x^2+3$
12. $f(x)=x^3-9x^2+27x$

Sketch a graph with the given characteristics.

13. $f(2)=f(4)=0$
    $f(3)$ is defined
    $f'(x)<0$, *if* $x<3$ (video)
    $f'(3)$ does not exist
    $f'(x)>0$, *if* $x>3$
    $f''(x)<0,\ x\neq 3$

14. $f(0)=f(2)=0$
    $f'(x)>0$ if $x<1$
    $f'(1)=0$
    $f'(x)<0$ if $x>1$
    $f''(x)<0$

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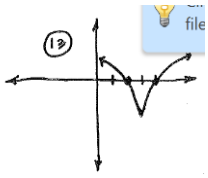
ANSWERS

1. CCUP: $(2,\infty)$
   CCDWN: $(-\infty,2)$
   POI: 2
2. CCUP: $\left(\frac{1}{2},\infty\right)$
   CCDWN: $\left(-\infty,\frac{1}{2}\right)$
   POI: $\frac{1}{2}$
3. CCUP: $\left(-\infty,\frac{-2\sqrt{3}}{3}\right)\left(\frac{2\sqrt{3}}{3},\infty\right)$
   CCDWN: $\left(\frac{-2\sqrt{3}}{3},\frac{2\sqrt{3}}{3}\right)$
   POI: $\frac{2\sqrt{3}}{3}$
4. CCUP: $(-\infty,\infty)$
   CCDWN: NONE
   POI: NONE
5. CCUP: $\left(-\sqrt{3},0\right)\left(\sqrt{3},\infty\right)$
   CCDWN: $\left(-\infty,\sqrt{3}\right)\left(0,\sqrt{3}\right)$
   POI: $\pm\sqrt{3},0$
6. CCUP: $(0,3)$
   CCDWN: $(3,\infty)$
   POI: 3
7. CCDWN: $\left(0,\frac{3\pi}{4}\right)\left(\frac{7\pi}{4},2\pi\right)$
   CCUP: $\left(\frac{3\pi}{4},\frac{7\pi}{4}\right)$
   POI: $\frac{3\pi}{4},\frac{7\pi}{4}$
8. 3 IS A REL MIN
   0 – TEST FAILS
9. -3/2 IS A REL MIN
10. 5 IS A REL MIN
11. 0 IS A REL MAX
    2 IS A REL MIN
12. AT 0 TEST FAILS!
    (THERE ARE NO REL EXTREMA)
13. 

14.