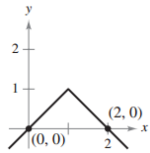


Rolles Theorem/Mean Value Theorem

1. In your own words, describe Rolles Theorem. Include a diagram.
2. In your own words, describe the Mean Value Theorem. Include a diagram.

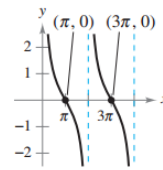
For 3-4, explain why Rolles Theorem does not apply even though there exists a and b such that $f(a) = f(b)$.

$$f(x) = 1 - |x - 1|$$



3.

$$f(x) = \cot \frac{x}{2}$$



4.

Determine for each if Rolles's Theorem can be applied. Find all values c , on the closed interval $[a, b]$ such that $f'(c) = 0$.

5. $f(x) = x^2 - 2x$; $[0, 2]$
6. $f(x) = x^2 - 5x + 4$; $[1, 4]$
7. $f(x) = (x - 3)(x + 1)^2$; $[-1, 3]$
8. $f(x) = x^{2/3} - 1$; $[-8, 8]$
9. $f(x) = \sin x$; $[0, 2\pi]$

Determine for each if the Mean Value Theorem can be applied. Find all values c , on the closed interval $[a, b]$ such that

$$f'(c) = \frac{f(b) - f(a)}{b - a}.$$

10. $f(x) = x^2$; $[-2, 1]$
11. $f(x) = x(x^2 - x - 2)$; $[-1, 1]$
12. $f(x) = x^{2/3}$; $[0, 1]$
13. $f(x) = \frac{x+1}{x}$; $\left[\frac{1}{2}, 2\right]$
14. $f(x) = \frac{x+1}{x}$; $[-2, 2]$
15. $f(x) = x - 2\sin x$; $[-\pi, \pi]$

Answers

1. -
2. -
3. $f(x)$ is NOT differentiable at $x = 1$.
4. $f(x)$ is NOT continuous at $x = 2\pi$.
5. Continuous, differential on $[0,2]$

$$f(0) = f(2) = 0$$

$$c = 1$$

6. Continuous, differential on $[1,4]$

$$f(1) = f(4) = 0$$

$$c = 5/2$$

7. Continuous, differential on $[-1,3]$

$$f(-1) = f(3) = 0$$

$$c = 5/3$$

8. $f(x)$ is NOT differentiable $x = 0$. So, Rolle's Thm does not apply.

9. Continuous, differential on $[0,2\pi]$

$$f(0) = f(2\pi) = 0$$

$$c = \frac{\pi}{2}, \frac{3\pi}{2}$$

10. $c = -\frac{1}{2}$

11. $c = -\frac{1}{3}$

12. $c = \frac{8}{27}$

13. $c = 1$

14. MVT does NOT apply, $f(x)$ is not continuous at $x = 0$.

15. $c = -\frac{\pi}{2}, \frac{\pi}{2}$