

**RIEMANN SUMS****FOLLOW ALONG**

T (TIME)	0	10	20	30	40	50	60
VELOCITY (M/S)	12.3	10.6	9.2	8.1	5.3	2.9	1.8

The velocity of windup toy car is recorded every 10 seconds in the table above. Estimate the distance traveled by the toy car using the Riemann sum indicated with 3 sub intervals.

MIDPOINT

LEFT HAND

RIGHT HAND

TRAPEZOIDAL RULE

Is the left hand approximation an OVERESTIMATE OR UNDERESTIMATE? Why?

Use the trapezoidal approximation to estimate the AVERAGE VELOCITY of the toy car.

The toy car manufacturer mathematicians have calculated that the velocity of their car is  $M(t) = 12(1.57)^{-0.05t}$ . Use the mathematician's formula to calculate the AVERAGE VELOCITY of the toy car over the 60 second period.

x	0	3	5	9
f(x)	8	12	15	16

Use the table to calculate each Riemann Sum with 3 subintervals

LEFT HAND

RIGHT HAND

TRAPEZOIDAL RULE

Is the trapezoidal approximation an OVERESTIMATE OR UNDERESTIMATE? Why?

**RIEMANN SUMS****SHOW THE WORK THAT LEADS TO YOUR ANSWER****YOU TRY!**

<b>Year</b>	<b>0</b>	<b>5</b>	<b>10</b>	<b>15</b>	<b>20</b>	<b>25</b>	<b>30</b>
<b>Rate</b>	<b>10.82</b>	<b>13.06</b>	<b>14.61</b>	<b>14.99</b>	<b>18.60</b>	<b>19.33</b>	<b>22.46</b>

Annual Coal Production in the United States, in quadrillion **BTU per year** is given in the table.

1. Use a midpoint Riemann Sum with three equal subdivisions to estimate coal produced during the 30-year period.

2. If  $R(t)$  is the rate of coal production  $t$  years since year 0, write an integral to represent the coal production over the 30-year period.

3. Write an expression for the average rate of coal production during the 30-year period.

4. Use a right hand Riemann Sum with 6 subintervals to estimate the coal produced between 1960 and 1990. Is this estimate an overestimate or an underestimate? Why?

5. Use a Trapezoidal approximation with 6 subintervals to estimate the coal produced during the 30-year period.

<b>x</b>	<b>0</b>	<b>2</b>	<b>4</b>	<b>6</b>
<b>f(x)</b>	<b>1</b>	<b>3</b>	<b>6</b>	<b>10</b>

6. Use the trapezoidal Rule to estimate  $\int_0^6 f(x)dx$ .  
Is this estimate an overestimate or an underestimate? Why?

<b>x</b>	<b>0</b>	<b>20</b>	<b>30</b>	<b>60</b>
<b>f(x)</b>	<b>8</b>	<b>12</b>	<b>16</b>	<b>10</b>

7. Use the table to estimate  $\int_0^{60} f(x)dx$  with

- The trapezoidal rule
- A left-hand Riemann Sum
- A right-hand Riemann Sum

<b>t (YEAR)</b>	<b>0</b>	<b>5</b>	<b>10</b>	<b>15</b>	<b>20</b>	<b>25</b>
<b>VALUE</b>	<b>\$225</b>	<b>\$448</b>	<b>\$905</b>	<b>\$1800</b>	<b>\$3675</b>	<b>\$7400</b>

The value  $B$  of a baseball card collection over a 25-year period starting in 1975 is given in the table. The value is in dollars  $t$  years after 1975.

8. Use the Trapezoidal Rule with 5 subintervals of equal length to estimate the AVERAGE VALUE of the baseball cards over the period from 1975-2000.

9. The given function  $B$  can be modeled by the function:  $B(t) = 225(1.15)^t$ . Use  $B(t)$  to find the approximate AVERAGE VALUE of the baseball card collection over the period from 1975-2000.

YOU TRY Answers:

1. 473.2 quadrillion BTU

2.  $\int_0^{30} R(t) dt$

3.  $\frac{1}{30} \int_0^{30} R(t) dt$

4. 515.25 quadrillion BTU

Coal Production Rate is increasing, so an OVER estimate

5. 486.15 quadrillion BTU

6. 29; OVER estimate since f is concave up

7. -

a. 730

b. 760

c. 700

8. \$2128.10

9. \$2055.43