

Review - Slope Fields part 1

Find the derivative of each of the following. 5

1. $g(x) = 3\arccos\left(\frac{x}{2}\right)$

2. $h(x) = \arctan\sqrt{x}$

3. $i(x) = x\arcsin(2x)$

4. $j(x) = (\arctan x)^2$

5. $k(x) = \operatorname{arcsec}(x^2)$

6. $l(x) = \frac{\arcsin x}{x}$

7. $m(x) = \cos(\arcsin x)$

Find the integral of each.

8. $\int_0^{\frac{\sqrt{3}}{2}} \frac{2}{1+4x^2} dx$

9. $\int \frac{1}{x\sqrt{4x^2-1}} dx$

10. $\int \frac{dx}{x^2+6x+13}$

11. $\int \frac{3dx}{1+16x^2}$

12. $\int \frac{dx}{3x^2+6x+6}$

13. $\int \frac{x-2}{\sqrt{9-x^2}} dx$

14. $\int \frac{x^3}{x^2-4} dx$

15. $\int \frac{2x-3}{x^2-6x+10} dx$

16. Find the equation of the tangent line at $x = 0$ for $g(x) = 3\arccos\left(\frac{x}{2}\right)$.

17. Consider the differential equation $\frac{dy}{dx} = xy$. Let $y = f(x)$ be the particular solution with the initial condition **$f(-1) = -2$** .

- Sketch the slope field on the grid.
- Write an equation of the tangent line at $x = -1$.
- Find the solution $y = f(x)$ to the given differential equation with the condition that **$f(-1) = -2$** .
- Use the tangent line to estimate the value of $f(-1.1)$
- Find the EXACT value of $f(-1.1)$. Is the estimate below or above the exact value? Why?



18. Consider the differential equation $\frac{dy}{dx} = y^2(2x+2)$. Let $y = f(x)$ be the particular solution to the differential equation with initial condition $f(0) = -1$.

- Find $\lim_{x \rightarrow 0} \frac{f(x)+1}{\sin x}$. Show all work that leads to your answer.
- Find $y = f(x)$, the particular solution to the differential equation with initial condition $f(0) = -1$.

Review - Slope Fields part 2

Solve each differential equation for y.

19. $\frac{dy}{dx} = \frac{5}{2y}$

20. $\frac{dy}{dx} = 2y\sqrt{x}$

21. $\frac{dy}{dx} + x - 3xy = 0$

22. $(1 - 4x^2)\frac{dy}{dx} - xy = 0$

23. $\frac{dy}{dp} = 3y^2tp^2$

Find the exponential equation in the form $y = Ce^{kt}$ that passes thru the following points.

24. (0,2) and (3,5)

25. (3,2), $(2, \frac{1}{2})$

Draw a slope field for each. Use the grids on the bottom of this sheet.

26. $\frac{dy}{dx} = x + y$

27. $\frac{dy}{dx} = -y^2$

28. $\frac{dy}{dx} = x - 1$

Simplify.

29. $\arcsin\left(-\frac{1}{2}\right)$

31. $\sec\left(\operatorname{arccot}\left(-\frac{4}{9}\right)\right)$

30. $\arctan\left(-\frac{\sqrt{3}}{3}\right)$

32. $\tan\left(\arcsin\frac{d}{c}\right)$

33. L'Hopitals Rule

a. $\lim_{x \rightarrow 0} \frac{2e^{3x} - 2}{x}$

f. $\lim_{x \rightarrow e} \frac{\ln x - 1}{x - e}$

k. $\lim_{x \rightarrow 2} \frac{x^2 + x - 6}{x^2 - 3x + 2}$

b. $\lim_{x \rightarrow \infty} \left(1 + \frac{3}{x}\right)^x$

g. $\lim_{x \rightarrow 0} \frac{\cos x - 1}{\cos 2x - 1}$

l. $\lim_{x \rightarrow 0} \frac{\arcsin \pi x}{x}$

c. $\lim_{x \rightarrow 1} \frac{\ln x}{x^2 - 1}$

h. $\lim_{x \rightarrow 0} \frac{e^x - e^{-x}}{\sin 3x}$

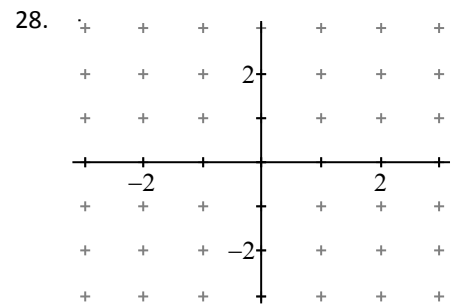
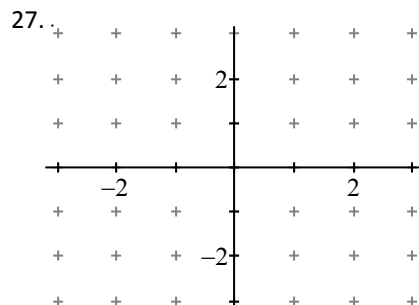
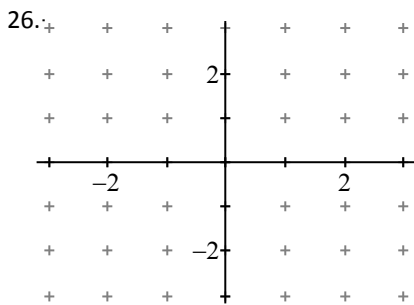
m. $\lim_{x \rightarrow 0} \frac{\sin \pi x}{2 \sin 5x}$

d. $\lim_{x \rightarrow \infty} x^2 \sin\left(\frac{1}{3x^2}\right)$

i. $\lim_{x \rightarrow \infty} \frac{e^x}{x^4}$

e. $\lim_{x \rightarrow 0^+} \sin x^{\sin x}$

j. $\lim_{x \rightarrow \infty} x^{1/x}$



Answers

1. $\frac{-3}{\sqrt{4-x^2}}$
2. $\frac{1}{2\sqrt{x(x+1)}}$
3. $\sin^{-1}(2x) + \frac{2x}{\sqrt{1-4x^2}}$
4. $\frac{2\tan^{-1}x}{x^2+1}$
5. $\frac{2}{x\sqrt{x^4-1}}$
6. $\frac{1}{x\sqrt{1-x^2}} - \frac{\sin^{-1}x}{x^2}$
7. $\frac{-x}{\sqrt{1-x^2}}$
8. $\frac{\pi}{3}$
9. $\text{arcsec}|2x| + c$
10. $\frac{1}{2}\tan^{-1}\left(\frac{x+3}{2}\right) + c$
11. $\frac{3}{4}\tan^{-1}(4x) + c$
12. $\frac{1}{3}\tan^{-1}(x+1) + c$
13. $-2\sin^{-1}\left(\frac{x}{3}\right) - \sqrt{9-x^2} + c$
14. $\frac{1}{2}x^2 + 2\ln|x^2-4| + c$
15. $\ln(x^2-6x+10) + 3\tan^{-1}(x-3) + c$
16. $y - \frac{3\pi}{2} = -\frac{3}{2}(x-0)$
17. -
 - a. -
 - b. $y+2=2(x+1)$
 - c. $y = -1.21306e^{\frac{1}{2}x^2}$
 - d. -2.2
 - e. -2.221
18. -
 - a. 2

$$\frac{dy}{dx} = y^2(2x+2)$$

$$\frac{dy}{y^2} = (2x+2) dx$$

$$\int \frac{dy}{y^2} = \int (2x+2) dx$$

$$-\frac{1}{y} = x^2 + 2x + C$$

$$-\frac{1}{-1} = 0^2 + 2 \cdot 0 + C \Rightarrow C = 1$$

$$-\frac{1}{y} = x^2 + 2x + 1$$

$$y = -\frac{1}{x^2 + 2x + 1} = -\frac{1}{(x+1)^2}$$
 - b.
19. $y^2 = 5x + c$

20. $y = Ce^{\frac{4}{3}x^2}$

21. $y = Ce^{\frac{3}{2}x^2} + \frac{1}{3}$

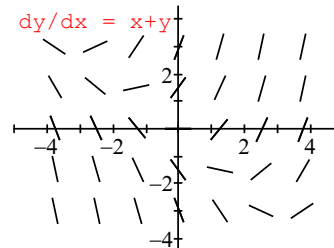
22. $y = C(1-4x^2)^{-\frac{1}{8}}$

23. $y = \frac{1}{C - tp^3}$

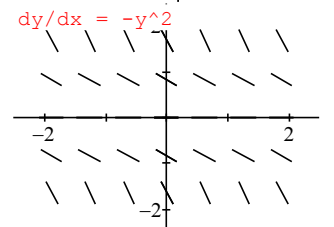
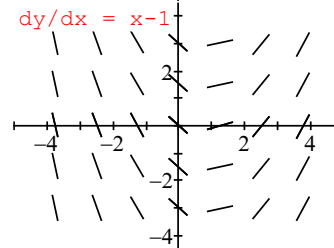
24. $y = 2e^{30543t}$

25. $y = \frac{1}{32}e^{t \ln 4}$

26. -



27.



28.

29. $-\pi/6$

30. $-\pi/6$

31. $\frac{-\sqrt{97}}{4}$

32. $\frac{d}{\sqrt{c^2-d^2}}$

33. -

- a. 6
- b. e^3
- c. $1/2$
- d. $1/3$
- e. 1
- f. $1/e$
- g. $1/4$
- h. $2/3$
- i. ∞
- j. 1
- k. 5
- l. π
- m. $\frac{\pi}{10}$