Review – Integrals
Calculus AB - Harter

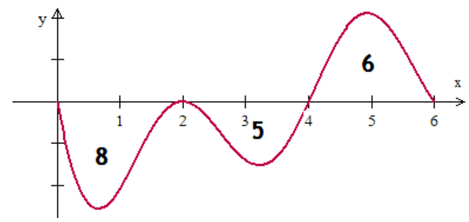
Find the integral of each.

1. $\int \frac{2-x}{\sqrt{x}}dx$
2. $\int \left(3x^3-2x^2-3\right)dx$
3. $\int -2\sin x\ dx$
4. $\int \left(\sec^2 x-\cos x\right)dx$
5. $\int \frac{2-x}{\sqrt[3]{x^5}}dx$
6. $\int \left(\csc^2 x\right)dx$

Evaluate each definite integral.

7. $\int_{-1}^{1} 2x^2 dx$
8. $\int_{0}^{\pi} \cos x\ dx$
9. $\int_{0}^{4} \sqrt{x}dx$
10. $\int_{-3}^{-1} \left(2-x^3\right)dx$
11. $\int_{1}^{9} \frac{2t^2+\sqrt{t^5}-1}{t^2}dt$
12. $\int_{1}^{4} \frac{x^2-2x}{\sqrt{x}}dx$

13. The graph of $f'(x)$ with labeled areas is shown. If $f(2)=1$, name the absolute minimum and absolute maximum on the interval [0,6]. Justify your answer.
14. Draw a picture of the integral represented by #8.



Find the average value of the function on the given interval.

15. $y=\cos x$, $[0,\pi]$
16. $y=1-x^3:[-1,1]$

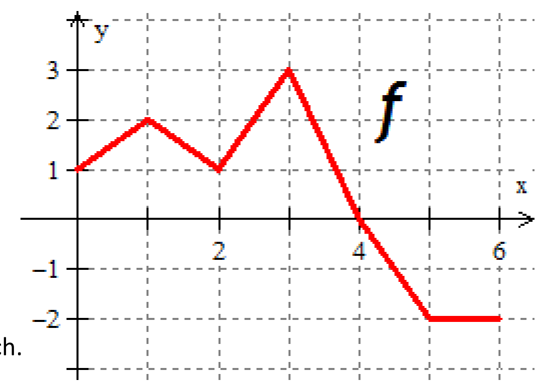
17. A car's speed is recorded in the chart.

t (in hours)	1	2	5	7	10	12
v(t) (in km/hr)	40	60	65	70	45	55

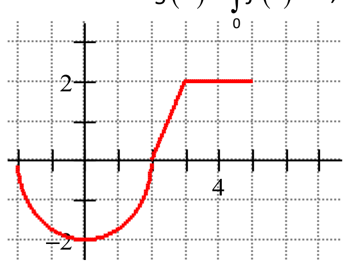
a. Use the trapezoidal approximation with **5 SUBINTERVALS** to estimate the distance traveled over the trip.
b. Use a right hand Riemann sum **5 SUBINTERVALS** to estimate the distance the car traveled.

18. Use the graph of f at the right if $g(x)=\int_0^x f(t)dt$.

a. Find $g(0), g(2), g(6)$
b. Where is g(x) at its maximum value?
c. Over what interval is g(x) decreasing?
d. Find $g'(2), g'(3), g'(6)$.
e. Find the average value of f on [0,4].



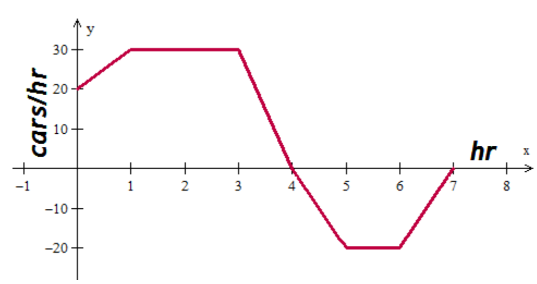
19. The graph of f on the interval [-2,5] is below. If $g(x)=\int_0^x f(t)\ dt$, find each.



a. g(0)
b. g(-2)
c. g(5)
d. g'(0)
e. g'(4)
f. g''(0)
g. g''(2.24)
h. Name the absolute minimum on [-2,5]. JYA.
i. Name the absolute maximum on [-2,5]. JYA.
j. Name the interval where g is concave up. JYA.
k. Name any point of inflection of [-2,5]. JYA.
l. If $h(x)=\int_{2x}^{4} f(t)\,dt$, find $h(1),\ h'(1),\ h''(1.25)$

20. The rates which cars are parked in a Houston parking lot are recorded versus time in the graph below starting at 7am. There are 60 cars in the lot at 9am.

a. How many cars are in the lot at 2pm?
b. What is the maximum amount of cars in the lot during the day and when does it occur?
c. How many cars were in the lot at 7 am?



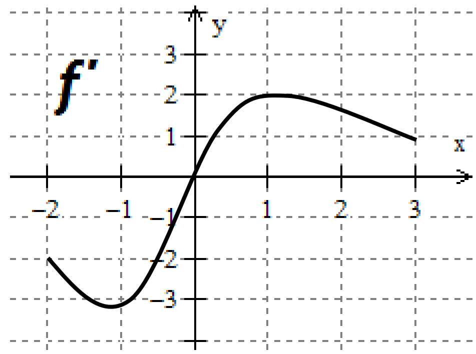
21. If $\int_{-1}^{3} f(x)dx = 5$, $\int_{3}^{7} f(x)dx = 10$ and $\int_{-1}^{2} f(x)dx = 2$ find each.

a. $\int_{2}^{3} f(x)dx$

b. $\int_{7}^{3} f(x)dx$

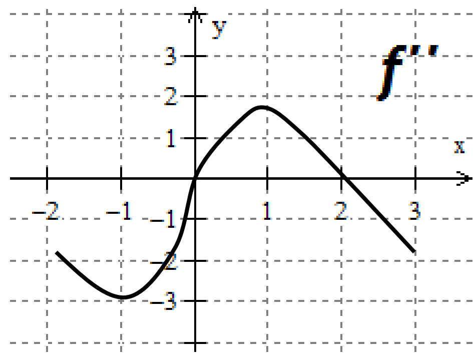
c. $\int_{2}^{7} f(x)dx$

22. Evaluate each: $\frac{d}{dx}\left[\int_{0}^{2x^2} \left(t^2 + 3t - 2\right)dt\right]$



23. Use the graph of f' to answer each.

a. Where is f increasing? JYA
b. Where is f concave up? JYA
c. Name the point(s) of inflection on f.
d. Name relative extrema on f.



24. Use the graph of f'' to answer each.

a. Where is f concave down? JYA
b. Where is f' concave down? JYA
c. Name the relative maximum on f'.
d. Name point(s) of inflection f.

25. Use your calculator if $f''(x) = 3\sqrt{\ln(x+1)} \cdot \cos 2x$ on the interval [0,3] then . . .

a. Where is f concave down?
b. Where is f′ concave down?
c. $f'''(1.256)$
d. $f''''(2.817)$

OPTIMIZATION

26. A pear orchard now has 28 trees per acre and an average yield of 450 pear per tree. For each additional tree planted the yield will decrease by 8 pears per tree. How many trees per acre will maximize the crop?

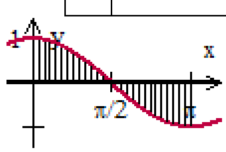
27. Your company needs design cylindrical metal containers with a volume of 15 cubic feet. The top and bottom are made of a sturdy material costing 5$ per square foot while the side can be made of a thinner material costing 2$ per square foot. Find the height, radius and cost of the least expensive container possible.

28. A rectangular field is fenced off along the bank of a river with no fence required along the river. The material for the fence costs 10$ per foot for the side parallel to the river and 6$ per foot for the sides perpendicular to the river. What should the dimensions of the enclosure be to maximize the area, if the total cost of the fence is 3800$?

ANSWERS

1. $4x^{\frac{1}{2}} - \frac{2}{3}x^{\frac{3}{2}} + C$
2. $\frac{3}{4}x^4 - \frac{2}{3}x^3 - 3x + C$
3. $2\cos x + C$
4. $\tan x - \sin x + C$
5. $-3x^{\frac{1}{3}} - \frac{3}{x^{\frac{2}{3}}} + C$
6. $-\cot x + C$
7. 4/3
8. 0
9. 16/3
10. 24
11. $32\frac{4}{9}$
12. $\frac{46}{15}$
13.

0	$g(0) = 1 + \int_2^0 f'(t)dt$	9	Absolute max is 9 at x = 0
2	$g(2) = 1 + \int_2^2 f'(t)dt$	1	
4	$g(4) = 1 + \int_2^4 f'(t)dt$	-4	Absolute min is -4 at x = 4
6	$g(6) = 1 + \int_2^6 f'(t)dt$	2	

14. 

15. 0
16. 1
17. –
    a. 645 km
    b. 640 km
18. –
    a. 0, 3, 3.5
    b. 4
    c. (4,6)
    d. 1, 3, -2
    e. 13/8
19. –
    a. 0
    b. π
    c. $5-\pi$
    d. -2
    e. 2
    f. 0
    g. 2
    h. Race:

x	$g(x) = \int_0^x f(t)dt$	$g(x)$	
-2	$g(-2) = \int_0^{-2} f(t)dt$	π	Absolute max is π at x=-2
2	$g(2) = \int_0^2 f(t)dt$	$-\pi$	Absolute min is $-\pi$ at x=2
5	$g(5) = \int_0^5 f(t)dt$	$5-\pi$	

    i. See h!
    j. $(0,3)$, Since $g'' = f'$, f is increasing on (0,3)
    k. x = 0, Since $g'' = f'$, f switches from decreasing to increasing at x=0
    l. $h(1) = 3,\ h'(1) = 0,\ h''(1.25) = -8$
20. –
    a. 65 cars
    b. 11 am, 105 cars
    c. 5 cars
21. –
    a. 3
    b. -10
    c. 13
22. $16x^5 + 24x^3 - 8x$
23. –
    a. (0,3), f' is positive
    b. (-1, 1), f' is increasing
    c. -1 and 1
    d. Rel min at x = 0
24. -
    a. (-2,0), (2,3) f'' is negative
    b. (-2, -1), (1,3), f'' is decreasing
    c. X = 2
    d. X = 0, 2
25. –
    a. (0.785, 2.35)
    b. (0.309, 1.620)
    c. –3.782
    d. –10.339
26. 42 trees
27. $R = 0.985,\ h = 4.924,\ \$91.39$
28. 190 (along river) by 158.333