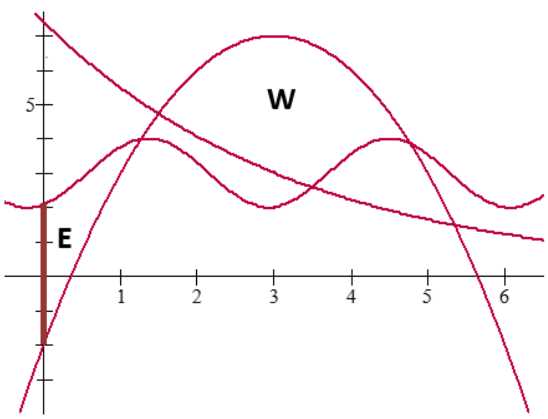
Review Area between Curves and Solids of Revolution
Calculus AB
Harter

* These problems can be done **WITHOUT** a calculator.

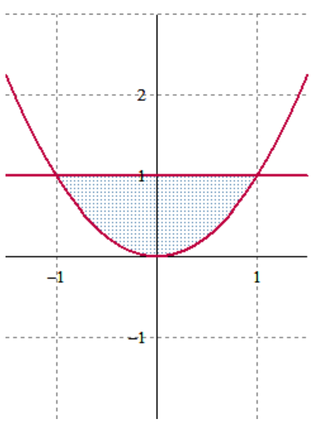
1. *Write the integral that would find the volume with a base enclosed by $y=x^2$, y= 0, x = 2 and slices that are:
   a. SQUARES
   b. SEMICIRCLES
   c. EQUILATERAL TRIANGLES
   d. RIGHT ISOSCELES TRIANGLES
2. * Find the area bounded by $f(x)=-2x+4$, $g(x)=x^2+3x+4$.
3. * Find the area between $y=x^2-2x$ and $y=x$ on the interval [0,4].
4. * Find the area between $y=e^{2x}$ and the x-axis on the interval [0,2].
5. * Find the area enclosed by $y=x^2$ and $y=x^3$.
6. *Consider the area R enclosed by $y=x^2$, $y=0$ and $y=-2x+8$. Set up, but do not solve the integral for each.
   a. The area of R.
   b. The volume when R is revolved over the x-axis.
   c. The volume when R is revolved over the y-axis.
   d. The volume when R is revolved over the x = 5.
7. Revolve the area enclosed by $y=2-x^2$, $y=x$, and the y-axis over the x-axis.
8. Revolve the area enclosed by $y=x^2-1$, $x=0$, and $y=3$ over the y-axis.
9. *Set up the integral only for the area revolved over the y = -1 that is enclosed by $y=\sqrt{x}$, $x=1$ and $x=2$.
10. * Write the integral that would find the volume with a circular base of $x^2+y^2=1$ and slices that are:
    a. SQUARES
    b. SEMICIRCLES
    c. EQUILATERAL TRIANGLES
    d. RIGHT ISOSCELES TRIANGLES



11. Consider the area W and E enclosed below by:

$$m(x)=-x^2+6x-2, \qquad u(x)=-\sin(2x+2)+3,$$
$$k(x)=e^{-0.3x+2}$$

   a. Find the area of region E.
   b. Find the volume of a solid with E as its base and SQUARE slices.
   c. Find the area of the region W.
   d. Find the volume when W is rotated over the x-axis.
   e. Find the volume when W is rotated over the y = 7.



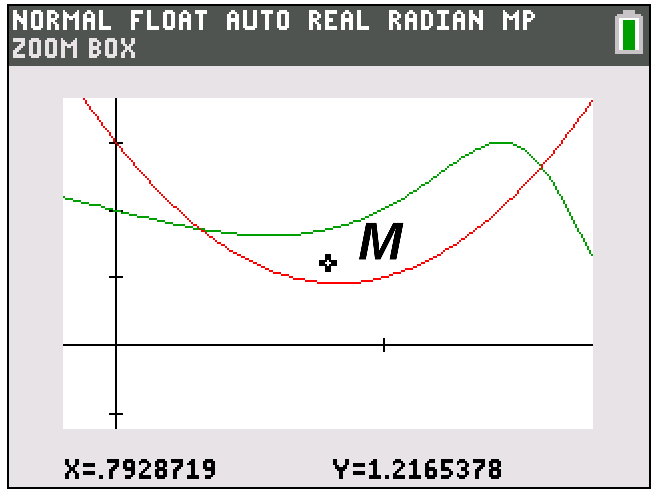
12. * Revolve the area enclosed by $y=x^2$ and $y=1$ (see fig at the right) about the following axis:
    a. $y=1$
    b. $y=-1$
    c. $y=2$

13. Find the area that is enclosed by $y=-x^3+2x^2+2x+3$ and $y=-x^2-5x+3$. Show integral(s) used.

14. Revolve the area enclosed by $x^2+y^2=9$, $y=1$, $y=3$ and $x=0$ in the first quadrant over the y-axis.

Consider the area M that is enclosed by the equations $f(x)=\sin\left(x^3-x\right)+2$, $g(x)=3x^2-5x+3$ shown at the right.

15. Find the area M.
16. Find the volume with RIGHT TRIANGLE slices on M, perpendicular to the x-axis.
17. Find the volume with EQULATERAL TRIANGLE slices on M, perpendicular to the x-axis.
18. M is rotated over the x-axis.
19. M is rotated over y = -2.
20. M is rotated over y-axis.
21. M is rotated over x = 3.



__Answers__

1. –
    a. 32/5
    b. $\frac{4}{5}\pi$
    c. $\frac{8\sqrt{3}}{5}$
    d. 16/5
2. 125/6
3. 19/3
4. $\frac{1}{2}\left(e^4-1\right)$
5. 1/12
6. –
    a. $\int_0^2 x^2\,dx + \int_2^4 (-2x+8)\,dx$
    b. $\pi\int_0^2 \left(x^2\right)^2\,dx + \pi\int_2^4 (-2x+8)^2\,dx$
    c. $\pi\int_0^4 \left(\frac{8-y}{2}\right)^2 - \sqrt{y}^2\,dy$
    d. $\pi\int_0^4 \left(5-\sqrt{y}\right)^2 - \left(5-\left(\frac{8-y}{2}\right)\right)^2\,dy$
7. $\frac{38\pi}{15}$
8. 8π
9. $\pi\int_1^2 \left[\left(1+\sqrt{x}\right)^2-1\right]dx$
10. -
    a. 16/3
    b. $\frac{2\pi}{3}$
    c. $\frac{4\sqrt{3}}{3}$
    d. 8/3
11. –
    a. 2.316
    b. 5.825
    c. 8.306
    d. 81.375π
    e. 34.915π
12. –
    a. 16pi/15
    b. 64pi/15
    c. 56pi/15
13. 62.750
14. 28pi/3
15. 0.984
16. 0.457
17. 0.396
18. 3.528pi
19. 7.463pi
20. 2.077pi
21. 3.826pi