

Optimization

1. What is the greatest possible volume for a right circular cone with slant height 10?
2. An apple orchard now has 30 trees per acre and the average yield is 400 apples per tree. For each additional tree planted per acre, the average yield is reduced by 10 apples. How many trees per acre will maximize the crop of apples?
3. A Boston lodge has asked the railroad company to run a special train to New York for its members. The railroad company agrees to run the train if at least 200 people will ride. The fare is to be \$8 per person if 200 people ride and will decrease \$.01 for all riders for every rider over 200. (Thus, if 250 people ride the train, the fare will be \$7.50 per rider). What number of passengers will give the railroad maximum revenue?
4. A rectangular field is to be fenced off along the bank of a river with no fence required along the river. If the material for the fence costs 4\$ per running foot for the 2 ends and 6\$ per running foot for the side parallel to the river, find the dimensions of the field of largest possible area that can be enclosed with \$1800.
5. A window, the shape of a rectangle capped with a semi-circle is to be surrounded by a 10 foot metal border. Find the radius of the semi-circular part if the total area of the window is to be maximum.
6. A company wants to develop cylindrical aluminum cans with a volume of 1000 cubic cm. What should the radius and height be to minimize the amount of aluminum used?
7. You are to make a metal canister of volume 100 cubic cm. Made of thicker metal on the top and bottom, the can top and bottom costs 5 cents per square cm. The sides costs 2 cents per square cm. Find the height and radius of a cylinder that minimizes the cost of producing the can.

8. A fence must be built to enclose an area of 20000 square feet. Fencing material costs 2\$ per foot for the two sides facing north and south and \$6 per square foot for the other two sides. Find the COST of the least expensive fence.
9. A fence must be built in a large field to enclose a rectangular area of 15625 square meters. One side of the area is bounded by an existing fence; no fence is needed there. Material for fencing costs 2\$ per meter for the two ends and 4\$ per meter for the side opposite the existing fence. Find the cost of the least expensive fence.
10. In planning a small restaurant, it is estimated that a profit of \$5 per seat will be made if the number of seats is between 60 and 80. On the other hand, the profit of each seat will decrease by 5 cents for each seat above 80. Find the number of seats that will maximize profit and the maximum profit.
11. Find the area of the largest isosceles triangle having a perimeter of 18 inches.
12. A closed box with a square base is to have a volume of 16000 cm cubed. The material for the top and bottom of the box costs 3\$ per square cm while the material for the sides costs \$1.50 per square cm. Find the dimensions that will lead to the minimum cost.
13. A cylindrical box will be tied up with a ribbon as shown in the figure. The longest piece of ribbon available is 130 cm. Ten cm is required to make the bow. Find the radius and the height of the box with largest possible volume.



14. Your company needs to design cylindrical metal containers with a volume of 16 cubic ft. The top and bottom are made of a sturdy material costing \$2 per square ft, while the material for the sides costs \$1 per square ft. Find the radius, height and the cost of the least expensive container.

Answers:

1. 403.067
2. 35
3. 500
4. 112.5 by 150
5. $10/(4+\pi)$
6. $r = 5.419$ $h = 10.84$
7. $r = 1.85$ $h = 9.27$
8. \$1959.59
9. \$1000
10. 90 seats \$405
11. $9\sqrt{3} \text{ in}^2$
12. 20 by 20 by 40
13. $r = 10 \text{ cm}$, $h = 10 \text{ cm}$
14. $r = 1.08$, $h = 4.34$, cost = \$44.11