REVIEW – LIMITS CALCULUS AB

Multiple Choice

1. Find $\lim_{x\to\infty}\frac{(1-2x^2)^3}{(x^3+1)^2}$
   a. 8
   b. 1
   c. 0
   d. $+\infty$
   e. -8

2. $\lim_{x\to -2^-}\frac{-|x+2|}{x+2}$
   a. -1
   b. 1
   c. $-\infty$
   d. $+\infty$
   e. Nonexistent

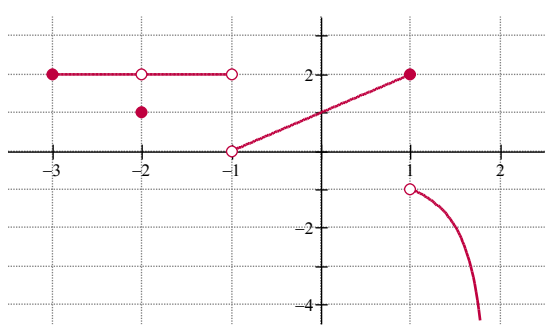
3. $\lim_{x\to 0}\frac{\sin^2 2x-2x^2}{x^2}$
   a. 0
   b. DNE
   c. -7/4
   d. 7/4
   e. 2

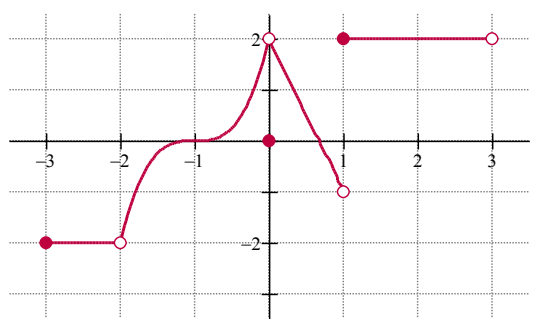
4. For $x\neq 0$, $\lim_{h\to 0}\frac{1}{h}\left(\frac{1}{x+h}-\frac{1}{x}\right)=$
   a. $\frac{1}{x^2}$
   b. $\frac{-2}{x}$
   c. $-\frac{1}{x^2}$
   d. $-\frac{2}{x^2}$
   e. 0

5. Find $\lim_{x\to -4^+}\frac{4x-6}{2x^2+5x-12}$
   a. 0
   b. 1
   c. ∞
   d. $-\infty$
   e. None of these

6. $\lim_{x\to 9}\frac{x-9}{3-\sqrt{x}}=$
   a. -6
   b. 6
   c. 0
   d. -12
   e. ∞

7. If $\lim_{n\to\infty}\frac{6n^2}{200-4n+kn^2}=\frac{1}{2}$, then k = ?
   a. 3
   b. 6
   c. 12
   d. 8
   e. 2





8. Use the graphs of f and g to answer each.
   a. f (-2)
   b. $\lim_{x\to -2} f(x)$
   c. $\lim_{x\to -1^-} g(x)$
   d. $\lim_{x\to 1^-} g(x)$
   e. $\lim_{x\to 0} f(x)+\lim_{x\to -2} g(x)$
   f. $\lim_{x\to 1^+} g(x)$
   g. $\lim_{x\to 1^-} g(x)$
   h. $\lim_{x\to -1^-}\frac{f(x)}{g(x)}$
   i. $\lim_{x\to 1} f(x)+g(x)$
   j. $\lim_{x\to 1^-} f(g(x))$
   k. $\lim_{x\to -1} g(f(x))$
   l. Name the open intervals that g is continuous.

Find each limit.

9. $\lim_{x\to -4} \frac{x^2-16}{x^3+64}$

10. $\lim_{x\to \infty} \frac{-3x^2+2x-1}{4-7x^2}$

11. $\lim_{x\to 0} \frac{2\tan 5x}{x}$

12. $\lim_{x\to \infty} \frac{3x^4-3x^2+2x-1}{4-5x^3}$

13. $\lim_{x\to 0} \frac{3\sin 4x}{x}$

14. $\lim_{x\to -2^-} \frac{1}{x+2}$

15. $\lim_{x\to 0} \frac{\frac{1}{\sqrt{1+x}}-1}{x}$

16. $\lim_{\Delta x\to 0} \frac{(x+\Delta x)^3-2(x+\Delta x)+3-(x^3-2x+3)}{\Delta x}$

17. $\lim_{x\to 5^-} \frac{|x-5|}{x-5}$

18. $\lim_{x\to 1} \frac{\sqrt{2x+1}-\sqrt{3}}{x-1}$

19. $\lim_{x\to -4} \frac{x}{(x+4)^2}$

20. $\lim_{x\to 0} \frac{1-\cos x}{3x^2}$

Name the discontinuities. Label as removable or non-removable.

21. $f(x)=\frac{2x-1}{3-7x}$

22. $f(x)=\frac{1-x}{x^2+2x-3}$

Find a and b or k so that f(x) is continuous.

23. $f(x)=\begin{cases} k\sin\frac{(x+3)\pi}{6}, & x\le 2 \\ \frac{3-\sqrt{11-x}}{x-2} & x>2 \end{cases}$

24. $f(x)=\begin{cases} \frac{2x-a}{4x^2-a^2}, & x\ne \frac{a}{2} \\ 12, & x=\frac{a}{2} \end{cases}$

25. $f(x)=\begin{cases} 3a-12xb, & x<-2 \\ 5, & x=-2 \\ ax^2+3xb, & x>-2 \end{cases}$

Perform the 3 step test for continuity on f (x) at given value of x. Use correct notation.

26. $f(x)=\begin{cases} 3\sin\frac{x}{2}; x\le 2\pi \\ -\tan\frac{x}{8}; x>2\pi \end{cases}$ AT x = 2π

27. $f(x)=\begin{cases} 3x^2-2, x<-1 \\ 1, x=-1 \\ 8x^3-5x+4, x>-1 \end{cases}$ AT x = -1

28. Consider: $f(x)=\frac{2-x}{2x^2-7x+6}$
    a. Name intervals where f(x) is continuous
    b. Label discontinuities as removable or non-removable.
29. Write your own example of the Intermediate Value Theorem.
30. Show that the IVT is valid on the given interval and find the value of c that is guaranteed by the theorem. $f(x)=x^3-x-4$ on [-1,5], $f(c)=2$

STUDY YOUR QUIZZES!!

Answers

1. E
2. B
3. E
4. C
5. C
6. A
7. C
8. –
   a. 1
   b. 2
   c. 0
   d. -1
   e. -1
   f. 2
   g. -1
   h. $-\infty$
   i. 1
   j. 0
   k. 2
   l. $(-3,-2)u(-2,0)u(0,1)u(1,3)$
9. -1/6
10. 3/7
11. 10
12. $-\infty$
13. 12
14. $-\infty$
15. -1/2
16. $3x^2-2$
17. -1
18. $\frac{\sqrt{3}}{3}$
19. $-\infty$
20. 1/6
21. NON - removable: x = 3/7
22. removable: x = 1
    non-removable: x = -3
23. k = 1/3
24. 1/24
25. a = 25/19, b = 5/114
26. 1. $f(2\pi)=3\sin\pi=0$
    2. $\lim_{x\to 2\pi^-} f(x)=0$, $\lim_{x\to 2\pi^+} f(x)=-1$
    3. -
    conclusion: sadly, f(x) is discontinuous at 2π
27. 1. $f(-1)=1$
    2. $\lim_{x\to -1^-} f(x)=1$, $\lim_{x\to -1^+} f(x)=-1$
    3. $\lim_{x\to -1} f(x)=1=f(-1)$
    conclusion: f(x) is continuous at -1
28. –
    a. $(-\infty,3/2)U(3/2,2)U(2,\infty)$
    b. Removable: x = 2
       Non-removable at x = 3/2
29. A runner accelerates from 0 to 8 mph in 2 seconds. Runner must have been traveling 4mph at least once between 0 and 2 seconds.
30. $f(-1)=-4,\ f(5)=116$ IVT applies since 2 is between -4 and 116. C=2