

1.

(a)  $a(2) = v'(2) = 1.587$  or  $1.588$

$$v(2) = -3 \sin(2) < 0$$

Speed is decreasing since  $a(2) > 0$  and  $v(2) < 0$ .

(b)  $v(t) = 0$  when  $\frac{t^2}{2} = \pi$

$$t = \sqrt{2\pi} \text{ or } 2.506 \text{ or } 2.507$$

Since  $v(t) < 0$  for  $0 < t < \sqrt{2\pi}$  and  $v(t) > 0$  for  $\sqrt{2\pi} < t < 3$ , the particle changes directions at  $t = \sqrt{2\pi}$ .

(c) Distance =  $\int_0^3 |v(t)| dt = 4.333$  or  $4.334$

(d)  $\int_0^{\sqrt{2\pi}} v(t) dt = -3.265$

$$x(\sqrt{2\pi}) = x(0) + \int_0^{\sqrt{2\pi}} v(t) dt = -2.265$$

Since the total distance from  $t = 0$  to  $t = 3$  is  $4.334$ , the particle is still to the left of the origin at  $t = 3$ . Hence the greatest distance from the origin is  $2.265$ .

$$2 : \begin{cases} 1 : a(2) \\ 1 : \text{speed decreasing} \\ \quad \text{with reason} \end{cases}$$

$$2 : \begin{cases} 1 : t = \sqrt{2\pi} \text{ only} \\ 1 : \text{justification} \end{cases}$$

$$3 : \begin{cases} 1 : \text{limits} \\ 1 : \text{integrand} \\ 1 : \text{answer} \end{cases}$$

$$2 : \begin{cases} 1 : \pm \text{ (distance particle travels} \\ \quad \text{while velocity is negative)} \\ 1 : \text{answer} \end{cases}$$

2.

(a)  $a(2) = v'(2) = -0.132$  or  $-0.133$

1 : answer

(b)  $v(2) = -0.436$

Speed is increasing since  $a(2) < 0$  and  $v(2) < 0$ .

1 : answer with reason

(c)  $v(t) = 0$  when  $\tan^{-1}(e^t) = 1$

$t = \ln(\tan(1)) = 0.443$  is the only critical value for  $y$ .

$v(t) > 0$  for  $0 < t < \ln(\tan(1))$

$v(t) < 0$  for  $t > \ln(\tan(1))$

$y(t)$  has an absolute maximum at  $t = 0.443$ .

3 :  $\left\{ \begin{array}{l} 1 : \text{sets } v(t) = 0 \\ 1 : \text{identifies } t = 0.443 \text{ as a candidate} \\ 1 : \text{justifies absolute maximum} \end{array} \right.$

(d)  $y(2) = -1 + \int_0^2 v(t) dt = -1.360$  or  $-1.361$

The particle is moving away from the origin since  $v(2) < 0$  and  $y(2) < 0$ .

4 :  $\left\{ \begin{array}{l} 1 : \int_0^2 v(t) dt \\ 1 : \text{handles initial condition} \\ 1 : \text{value of } y(2) \\ 1 : \text{answer with reason} \end{array} \right.$

3.

$$(a) \int_0^{24} v(t) dt = \frac{1}{2}(4)(20) + (12)(20) + \frac{1}{2}(8)(20) = 360$$

The car travels 360 meters in these 24 seconds.

2 :  $\begin{cases} 1 : \text{value} \\ 1 : \text{meaning with units} \end{cases}$

(b)  $v'(4)$  does not exist because

$$\lim_{t \rightarrow 4^-} \left( \frac{v(t) - v(4)}{t - 4} \right) = 5 \neq 0 = \lim_{t \rightarrow 4^+} \left( \frac{v(t) - v(4)}{t - 4} \right).$$

$$v'(20) = \frac{20 - 0}{16 - 24} = -\frac{5}{2} \text{ m/sec}^2$$

3 :  $\begin{cases} 1 : v'(4) \text{ does not exist, with explanation} \\ 1 : v'(20) \\ 1 : \text{units} \end{cases}$

$$(c) a(t) = \begin{cases} 5 & \text{if } 0 < t < 4 \\ 0 & \text{if } 4 < t < 16 \\ -\frac{5}{2} & \text{if } 16 < t < 24 \end{cases}$$

$a(t)$  does not exist at  $t = 4$  and  $t = 16$ .

2 :  $\begin{cases} 1 : \text{finds the values } 5, 0, -\frac{5}{2} \\ 1 : \text{identifies constants with correct intervals} \end{cases}$

(d) The average rate of change of  $v$  on  $[8, 20]$  is

$$\frac{v(20) - v(8)}{20 - 8} = -\frac{5}{6} \text{ m/sec}^2.$$

No, the Mean Value Theorem does not apply to  $v$  on  $[8, 20]$  because  $v$  is not differentiable at  $t = 16$ .

2 :  $\begin{cases} 1 : \text{average rate of change of } v \text{ on } [8, 20] \\ 1 : \text{answer with explanation} \end{cases}$

4.

(a)  $a(4) = v'(4) = \frac{5}{7}$

(b)  $v(t) = 0$

$$t^2 - 3t + 3 = 1$$

$$t^2 - 3t + 2 = 0$$

$$(t-2)(t-1) = 0$$

$$t = 1, 2$$

$$v(t) > 0 \text{ for } 0 < t < 1$$

$$v(t) < 0 \text{ for } 1 < t < 2$$

$$v(t) > 0 \text{ for } 2 < t < 5$$

The particle changes direction when  $t = 1$  and  $t = 2$ .

The particle travels to the left when  $1 < t < 2$ .

(c)  $s(t) = s(0) + \int_0^t \ln(u^2 - 3u + 3) du$

$$s(2) = 8 + \int_0^2 \ln(u^2 - 3u + 3) du$$

$$= 8.368 \text{ or } 8.369$$

(d)  $\frac{1}{2} \int_0^2 |v(t)| dt = 0.370 \text{ or } 0.371$

1 : answer

3 :  $\left\{ \begin{array}{l} 1 : \text{sets } v(t) = 0 \\ 1 : \text{direction change at } t = 1, 2 \\ 1 : \text{interval with reason} \end{array} \right.$

3 :  $\left\{ \begin{array}{l} 1 : \int_0^2 \ln(u^2 - 3u + 3) du \\ 1 : \text{handles initial condition} \\ 1 : \text{answer} \end{array} \right.$

2 :  $\left\{ \begin{array}{l} 1 : \text{integral} \\ 1 : \text{answer} \end{array} \right.$