1

(OZ, ex. 26, p.

The graph of the derivative of a function f is shown below. Use the graph of f' to answer the following questions about f.

(a) On which intervals is f increasing? Decreasing?

(b) Where does f have a stationary point?

(c) Where does f have a local maximum? A local minimum?

(d) On which intervals is the graph of f concave up? Concave down?

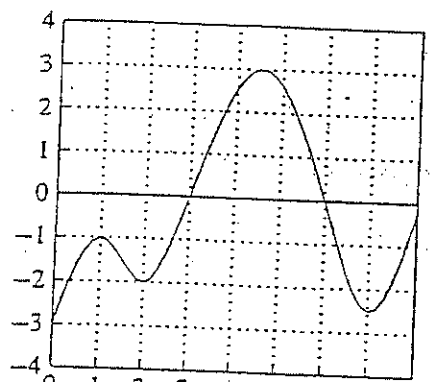
(e) Where does f have a point of inflection?

(f) Where does f achieve its maximum value on the interval $[0, 2]$? Its minimum value?

(g) Where does f achieve its maximum value on the interval $[3, 6]$? Its minimum value?

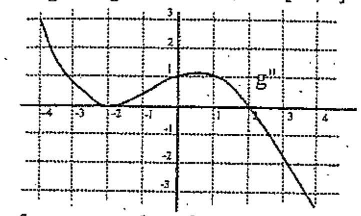
(h) Assume $f(0) = 0$. Sketch a graph of f.

(i) How does your answer to part (h) change if $f(0) = 5$?



Graph of f'

2 The graph of the second derivative of g is shown below. Use the graph to answer the following questions about g and g' on the interval $[-4, 4]$.

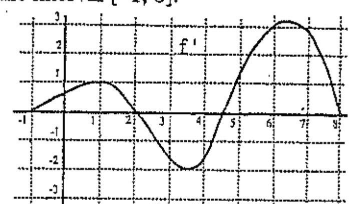


a) Where is the graph of g concave down?

b) Where does g have points of inflection?

c) Suppose that $g'(0) = 0$. Is g increasing or decreasing at $x = 2$? Justify your answer briefly.

3 The graph of the derivative of the function f is shown below. Use the graph to answer questions about f on the interval $[-1, 8]$.



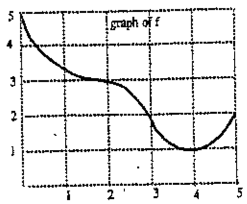
a) Suppose that $f(1) = 5$. Find an equation of the tangent line to the graph of f at the point (1, 5).

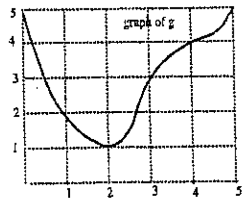
b) On what intervals is f both increasing and concave down?

c) Where does f have a local minimum?

d) Assume that $f(0) = 0$. Sketch a possible graph of f.

4. The function h is defined by $h(x) = f[g(x)]$, where f and g are functions whose graphs are shown below.





a) Evaluate $h(2)$.

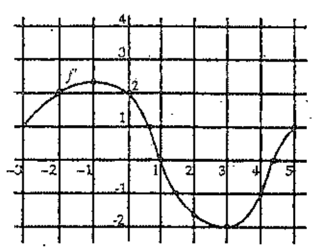
b) Estimate $h'(1)$.

c) Is the graph of the composite function h increasing or decreasing at $x = 3$? Show your reasoning.

5. a) The function f has a derivative defined by the rule $f'(x) = x^3 - 4.5x^2 - 12x - 5$. Determine the intervals on which the graph of the original function f is concave up.

b) Determine the value of k so that $f(x) = x^3 - kx^2 + 2$ will have a point of inflection at $x = 1$.

6. The graph of the derivative f' of a function f on the closed interval $[-3, 5]$ is shown below. With this graph as an aid, answer each of the following questions.



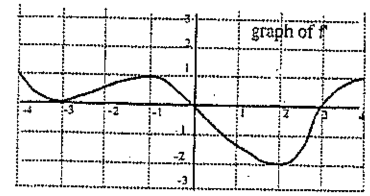
a) What are the critical numbers of f ?

b) Determine the x-coordinate(s) of any local minimum point(s) of f.

c) On what interval is f both increasing and concave down?

d) Let another function g be defined by $g(x) = x^2 - 3x - 1$. With this information and the graph of f above, determine the value of the derivative of the composition $y = f(g(x))$ at $x = 3$.

7. The graph of the derivative of the function f is shown below.



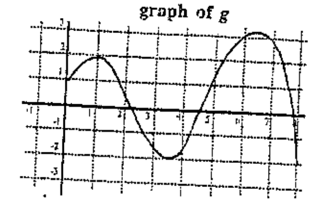
a) Suppose $f(-4) = 0$. Find an equation for the tangent line to the graph of f at $(-4, 0)$.

b) Where does the graph of f have a local minimum? Justify briefly.

c) On what intervals, if any, is the graph of f concave down?

d) If $f(-4) = 0$, sketch a possible graph of f.

8. Use the graph of g to estimate the numbers in $[0, 8]$ that satisfy the conclusion of the Mean Value Theorem.



9. Let $f(x) = x^3 + px^2 + qx$.

a) Find the values of p and q so that $f(-1) = -8$ and $f'(-1) = 12$.

b) Find the value of p so that the graph of f changes concavity at $x = 2$.