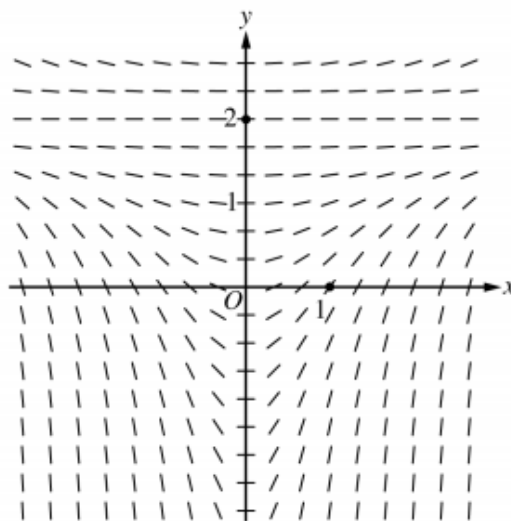


CALCULUS AB - HARTER

6. Consider the differential equation $\frac{dy}{dx} = \frac{1}{3}x(y-2)^2$.

1

- (a) A slope field for the given differential equation is shown below. Sketch the solution curve that passes through the point $(0, 2)$, and sketch the solution curve that passes through the point $(1, 0)$.



- (b) Let $y = f(x)$ be the particular solution to the given differential equation with initial condition $f(1) = 0$. Write an equation for the line tangent to the graph of $y = f(x)$ at $x = 1$. Use your equation to approximate $f(0.7)$.
- (c) Find the particular solution $y = f(x)$ to the given differential equation with initial condition $f(1) = 0$.
-

2

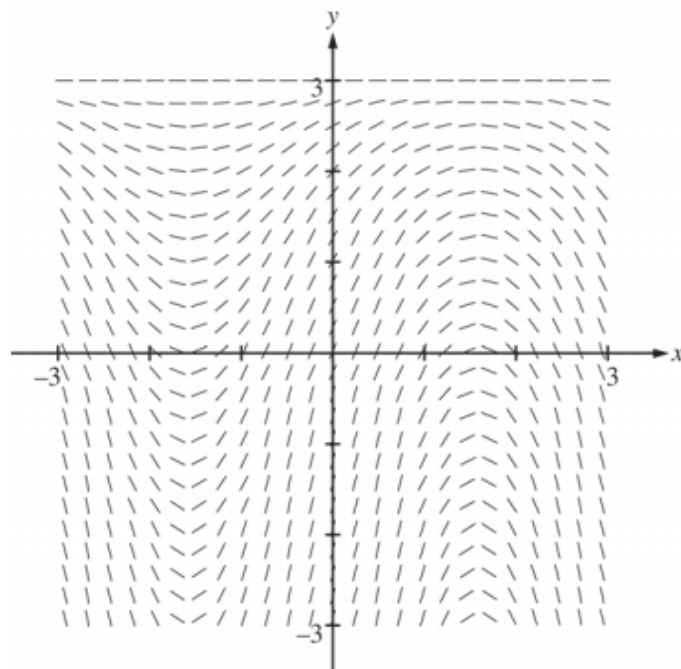
Consider the differential equation $\frac{dy}{dx} = \frac{y^2}{x-1}$.

- (a) On the axes provided, sketch a slope field for the given differential equation at the six points indicated.
- (b) Let $y = f(x)$ be the particular solution to the given differential equation with the initial condition $f(2) = 3$. Write an equation for the line tangent to the graph of $y = f(x)$ at $x = 2$. Use your equation to approximate $f(2.1)$.
- (c) Find the particular solution $y = f(x)$ to the given differential equation with the initial condition $f(2) = 3$.
-

3

6. Consider the differential equation $\frac{dy}{dx} = (3 - y)\cos x$. Let $y = f(x)$ be the particular solution to the differential equation with the initial condition $f(0) = 1$. The function f is defined for all real numbers.

- (a) A portion of the slope field of the differential equation is given below. Sketch the solution curve through the point $(0, 1)$.



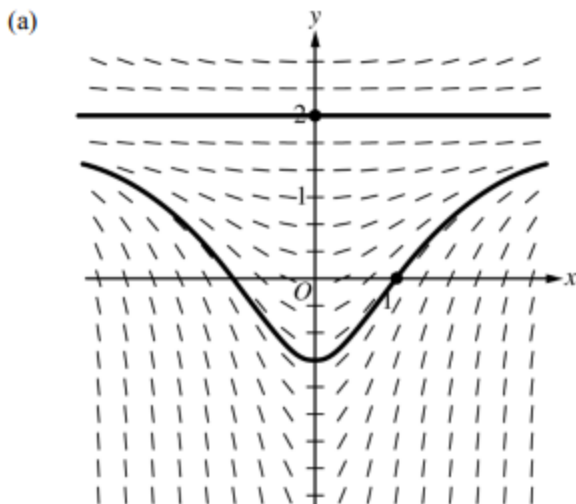
- (b) Write an equation for the line tangent to the solution curve in part (a) at the point $(0, 1)$. Use the equation to approximate $f(0.2)$.
- (c) Find $y = f(x)$, the particular solution to the differential equation with the initial condition $f(0) = 1$.
-

4

Consider the differential equation $\frac{dy}{dx} = e^y(3x^2 - 6x)$. Let $y = f(x)$ be the particular solution to the differential equation that passes through $(1, 0)$.

- (a) Write an equation for the line tangent to the graph of f at the point $(1, 0)$. Use the tangent line to approximate $f(1.2)$.
- (b) Find $y = f(x)$, the particular solution to the differential equation that passes through $(1, 0)$.
-

1



(b) $\left. \frac{dy}{dx} \right|_{(x,y)=(1,0)} = \frac{4}{3}$

An equation for the line tangent to the graph of $y = f(x)$ at

$x = 1$ is $y = \frac{4}{3}(x - 1)$.

$f(0.7) \approx \frac{4}{3}(0.7 - 1) = -0.4$

(c) $\frac{dy}{dx} = \frac{1}{3}x(y - 2)^2$

$\int \frac{dy}{(y - 2)^2} = \int \frac{1}{3}x \, dx$

$\frac{-1}{y - 2} = \frac{1}{6}x^2 + C$

$\frac{1}{2} = \frac{1}{6} + C \Rightarrow C = \frac{1}{3}$

$\frac{-1}{y - 2} = \frac{1}{6}x^2 + \frac{1}{3} = \frac{x^2 + 2}{6}$

$y = 2 - \frac{6}{x^2 + 2}$

Note: this solution is valid for $-\infty < x < \infty$.

2 : $\begin{cases} 1 : \text{solution curve through } (0, 2) \\ 1 : \text{solution curve through } (1, 0) \end{cases}$

Curves must go through the indicated points, follow the given slope lines, and extend to the boundary of the slope field.

2 : $\begin{cases} 1 : \text{equation of tangent line} \\ 1 : \text{approximation} \end{cases}$

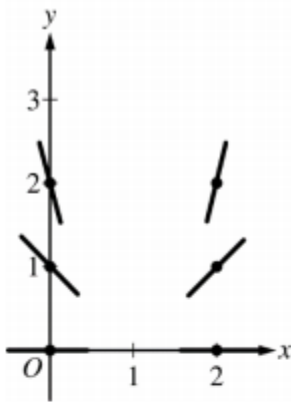
5 : $\begin{cases} 1 : \text{separation of variables} \\ 2 : \text{antiderivatives} \\ 1 : \text{constant of integration} \\ \text{and uses initial condition} \\ 1 : \text{solves for } y \end{cases}$

Note: 0/5 if no separation of variables

Note: max 3/5 [1-2-0-0] if no constant of integration

2

(a)



$$(b) \left. \frac{dy}{dx} \right|_{(x,y)=(2,3)} = \frac{3^2}{2-1} = 9$$

An equation for the tangent line is $y = 9(x - 2) + 3$.

$$f(2.1) \approx 9(2.1 - 2) + 3 = 3.9$$

$$(c) \frac{1}{y^2} dy = \frac{1}{x-1} dx$$

$$\int \frac{1}{y^2} dy = \int \frac{1}{x-1} dx$$

$$-\frac{1}{y} = \ln|x-1| + C$$

$$-\frac{1}{3} = \ln|2-1| + C \Rightarrow C = -\frac{1}{3}$$

$$-\frac{1}{y} = \ln|x-1| - \frac{1}{3}$$

$$y = \frac{1}{\frac{1}{3} - \ln(x-1)}$$

Note: This solution is valid for $1 < x < 1 + e^{1/3}$.

$$2 : \begin{cases} 1 : \text{zero slopes} \\ 1 : \text{nonzero slopes} \end{cases}$$

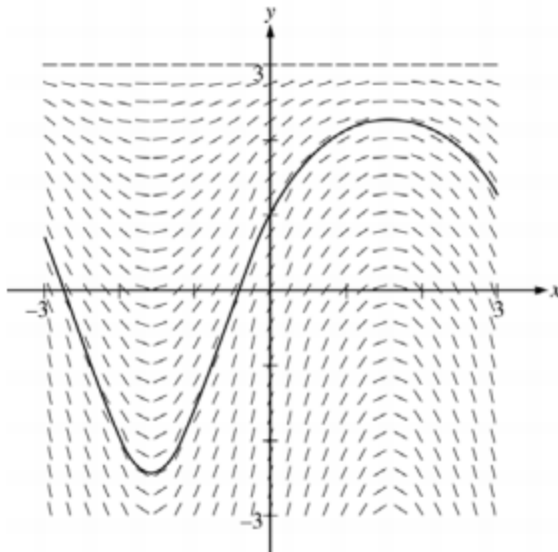
$$2 : \begin{cases} 1 : \text{tangent line equation} \\ 1 : \text{approximation} \end{cases}$$

$$5 : \begin{cases} 1 : \text{separation of variables} \\ 2 : \text{antiderivatives} \\ 1 : \text{constant of integration and} \\ \quad \text{uses initial condition} \\ 1 : \text{solves for } y \end{cases}$$

Note: max 3/5 [1-2-0-0] if no constant of integration

Note: 0/5 if no separation of variables

3 (a)



(b) $\left. \frac{dy}{dx} \right|_{(x,y)=(0,1)} = 2 \cos 0 = 2$

An equation for the tangent line is $y = 2x + 1$.

$f(0.2) \approx 2(0.2) + 1 = 1.4$

(c) $\frac{dy}{dx} = (3 - y) \cos x$

$\int \frac{dy}{3 - y} = \int \cos x \, dx$

$-\ln|3 - y| = \sin x + C$

$-\ln 2 = \sin 0 + C \Rightarrow C = -\ln 2$

$-\ln|3 - y| = \sin x - \ln 2$

Because $y(0) = 1$, $y < 3$, so $|3 - y| = 3 - y$

$3 - y = 2e^{-\sin x}$

$y = 3 - 2e^{-\sin x}$

Note: this solution is valid for all real numbers.

1 : solution curve

2 : $\left\{ \begin{array}{l} 1 : \text{tangent line equation} \\ 1 : \text{approximation} \end{array} \right.$

6 : $\left\{ \begin{array}{l} 1 : \text{separation of variables} \\ 2 : \text{antiderivatives} \\ 1 : \text{constant of integration} \\ 1 : \text{uses initial condition} \\ 1 : \text{solves for } y \end{array} \right.$

Note: max 3/6 [1-2-0-0-0] if no constant of integration

Note: 0/6 if no separation of variables

4

$$(a) \left. \frac{dy}{dx} \right|_{(x,y)=(1,0)} = e^0 (3 \cdot 1^2 - 6 \cdot 1) = -3$$

An equation for the tangent line is $y = -3(x - 1)$.

$$f(1.2) \approx -3(1.2 - 1) = -0.6$$

$$(b) \frac{dy}{e^y} = (3x^2 - 6x) dx$$

$$\int \frac{dy}{e^y} = \int (3x^2 - 6x) dx$$

$$-e^{-y} = x^3 - 3x^2 + C$$

$$-e^{-0} = 1^3 - 3 \cdot 1^2 + C \Rightarrow C = 1$$

$$-e^{-y} = x^3 - 3x^2 + 1$$

$$e^{-y} = -x^3 + 3x^2 - 1$$

$$-y = \ln(-x^3 + 3x^2 - 1)$$

$$y = -\ln(-x^3 + 3x^2 - 1)$$

Note: This solution is valid on an interval containing $x = 1$ for which $-x^3 + 3x^2 - 1 > 0$.

$$3 : \begin{cases} 1 : \frac{dy}{dx} \text{ at the point } (x, y) = (1, 0) \\ 1 : \text{tangent line equation} \\ 1 : \text{approximation} \end{cases}$$

$$6 : \begin{cases} 1 : \text{separation of variables} \\ 2 : \text{antiderivatives} \\ 1 : \text{constant of integration} \\ 1 : \text{uses initial condition} \\ 1 : \text{solves for } y \end{cases}$$

Note: max 3/6 [1-2-0-0-0] if no constant of integration

Note: 0/6 if no separation of variables