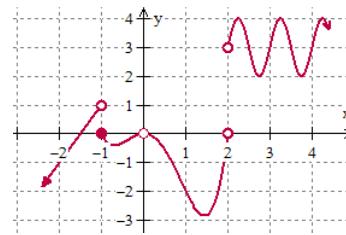


8 - Limits and Continuity

Calculus AB

1. Answer each about the graph of $f(x)$.
- Name all removable discontinuities.
 - Name all non-removable discontinuities.
 - Name the open intervals that f is continuous.



2. Use the 3-step test to find if f is continuous at the given value of x .

a. $f(x) = \begin{cases} 2-x^3 & ; x < -2 \\ 4x+3 & ; x > -2 \end{cases}$

b. $f(x) = \begin{cases} 3\sin 2x & ; x \geq \frac{\pi}{4} \\ \frac{16x}{\pi} - 1 & ; x < \frac{\pi}{4} \end{cases}$

c. $Q(x) = \begin{cases} 2x^3 - 3 & ; x < -1 \\ -4 & ; x = -1 \\ -3 - \sqrt{3-x} & ; x > -1 \end{cases}$

3. Find the discontinuities. Label as REMOVABLE OR NON-REMOVABLE.

a. $g(x) = \begin{cases} 8x^2 - 10x + 3 & \\ 2x^2 + 3x - 2 & \end{cases}$

b. $T(x) = \begin{cases} 3x - 2 & ; x > 1 \\ -2\cos\left(\frac{2\pi x}{3}\right) & ; x < 1 \end{cases}$

4. Evaluate by any method. These should all be QUICK LIMITS!

a. $\lim_{x \rightarrow 2^+} \frac{1}{x-2}$

f. $\lim_{x \rightarrow -3^+} \frac{x^2 - 9}{x^3 + 27}$

k. $\lim_{x \rightarrow 0^-} \frac{\sin x}{2x^2 - x}$

b. $\lim_{x \rightarrow \frac{\pi}{4}} 2\tan 3x \cdot \sin 5x$

g. $\lim_{x \rightarrow \frac{\pi}{6}} \sin 7x \cdot \cos 5x$

l. $\lim_{x \rightarrow 0} \frac{\sin^2(2x)}{x^2}$

c. $\lim_{x \rightarrow 4^-} \frac{|x-4|}{x-4}$

h. $\lim_{x \rightarrow -2^+} \frac{x+2}{x^2 - 4}$

m. $\lim_{x \rightarrow 0} \frac{x + \sin x}{x}$

d. $\lim_{x \rightarrow 0^+} \frac{\sin 4x}{\tan 8x}$

i. $\lim_{x \rightarrow 3^+} \frac{\frac{1}{x} - \frac{1}{3}}{x-3}$

e. $\lim_{x \rightarrow 2^-} \frac{1}{x-2}$

j. $\lim_{x \rightarrow -2} \frac{x}{x-2}$

5. What is the Intermediate Value Theorem? Make up your own story that exemplifies the IVT.

6. Use the IVT to explain why each function has a zero on the given interval.

a. $f(x) = \frac{1}{16}x^4 - x^3 + 3, [1, 2]$

c. $f(x) = x^2 - 2 - \cos x, [0, \pi]$

b. $f(x) = x^3 + 3x - 2, [0, 1]$

7. Verify that the IVT applies to the given interval and find the value of c guaranteed by the theorem.

a. $f(x) = x^2 + x - 1; [0, 5], f(c) = 11$

c. $f(x) = \frac{x^2 + x}{x-1}; \left[\frac{5}{2}, 4\right], f(c) = 6$

b. $f(x) = x^2 - 6x + 8; [0, 3], f(c) = 0$

8. $g(x) = \begin{cases} x^2 - 4 & ; \text{if } x \geq -1 \\ 2x - 3 & ; \text{if } x < -1 \end{cases}$

a. Sketch $g(x)$

d. $\lim_{x \rightarrow -1^+} g(x)$

b. $g(-1)$

e. $\lim_{x \rightarrow -1} g(x)$

c. $\lim_{x \rightarrow -1^-} g(x)$

f. is $g(x)$ continuous at $x = -1$

9. What value should you assign to "a" or 'b' to make f continuous?

a. $f(x) = \begin{cases} x^2 - 1; & x < 3 \\ 2ax; & x \geq 3 \end{cases}$

b. $f(x) = \begin{cases} ax^2 - 2b; & x < 1 \\ 5; & x = 1 \\ 3a - 4bx; & x > 1 \end{cases}$

c. $f(x) = \begin{cases} \frac{4x^2 - a^2}{2x - a}; & x \neq \frac{a}{2} \\ 12; & x = \frac{a}{2} \end{cases}$

d. $f(x) = \begin{cases} \sin ax; & x < 1 \\ x - \frac{3}{2}; & x \geq 1 \end{cases}$

10. Find the limits from the graph of f on the interval [-4, 4].

a. $\lim_{x \rightarrow -2} f(x)$

b. $\lim_{x \rightarrow -1^+} f(x)$

c. $\lim_{x \rightarrow -1^-} f(x)$

d. $\lim_{x \rightarrow -1} f(x)$

e. $\lim_{x \rightarrow 2^-} f(x)$

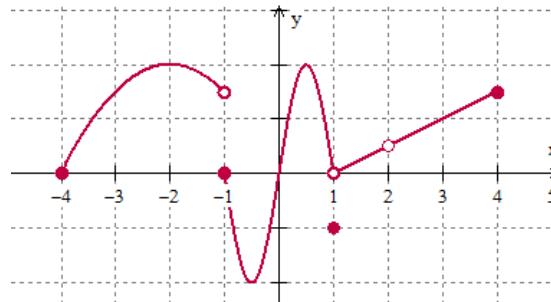
f. $\lim_{x \rightarrow 2^+} f(x)$

g. $\lim_{x \rightarrow 2} f(x)$

h. $f(-1)$

i. $f(2)$

j. Name the largest open intervals that f is continuous.



ANSWERS

1. -

- a. $x = 0$
 b. $x = -1, 2$
 c. $(-\infty, -1) \cup (-1, 0) \cup (0, 2) \cup (2, \infty)$

2.

a. Step 1: $f(-2) = \text{DNE}!$

(1A)

Conclusion: $f(x)$ is
NOT CONTINUOUS AT $x = -2$.

b. (1B)

Step 1: $f(\pi/4) = 3 \sin \frac{\pi}{4} = 3$

Step 2: $\lim_{x \rightarrow \pi/4^+} f(x) = 3$
 $\lim_{x \rightarrow \pi/4^-} f(x) = \frac{16 \cdot \pi/4}{\pi} - 1 = 4 - 1 = 3$

Step 3: $f(\pi/4) = 3 = \lim_{x \rightarrow \pi/4} f(x)$

Conclusion: $f(x)$ is CONTINUOUS AT $x = \pi/4$.

c. (1C)

Step 1: $Q(-1) = -4$

Step 2

$$\lim_{x \rightarrow -1^+} Q(x) = -3 - \sqrt{3} + 1 = -5$$

$$\lim_{x \rightarrow -1^-} Q(x) = 2(-1)^3 - 3 = -5$$

Step 3

$$Q(-1) = -4 \neq \lim_{x \rightarrow -1} Q(x) = -5$$

Conclusion:

$Q(x)$ is NOT CONTINUOUS AT $x = -1$!

3. -

- a. Rem at $x=1/2$, non-Rem at $x=-2$
 b. Rem at $x=1$

4. -

- a. ∞
 b. $\sqrt{2}$
 c. -1
 d. $\frac{1}{2}$
 e. $-\infty$
 f. $-2/9$
 g. $\frac{\sqrt{3}}{4}$
 h. $-1/4$
 i. $1/9$

j. $\frac{1}{2}$

k. -1

l. 4

m. 2

5. Look in your notes!

6. -

a. $f(1) = 2 \frac{1}{16}$

b. $f(2) = -4$

There must be at least one zero since zero is between $2 \frac{1}{16}$ and -4.

c. $f(0) = -2$

d. $f(1) = 2$

There must be at least one zero since zero is between -2 and 2.

e. $f(0) = -3$

f. $f(\pi) = \pi^2 - 1$

There must be at least one zero since zero is between -3 and $\pi^2 - 1$.

7. -

- a. $f(x)$ is cont. on $[0, 5]$; $f(0) = -1, f(5) = 29$ and $-1 < 11 < 29$; $c = 3$
 b. $f(x)$ is cont. on $[0, 3]$; $f(0) = 8, f(3) = -1$ and $1 < 0 < 8$; $c = 2$
 c. $f(x)$ is cont. on $[5/2, 4]$; $f(5/2) = 35/6, f(4) = 20/3$ and $35/6 < 6 < 20/3$; $c = 3$

8. -

- a. -
 b. -3
 c. -5
 d. -3
 e. dne
 f. no

9. -

- a. $a = 4/3$
 b. $a = -5, b = -5$
 c. $a = 6$
 d. $a = \frac{7\pi}{6}, \frac{11\pi}{6} \pm 2\pi n$; n is an counting number

10. -

- a. 2
 b. 0
 c. 1.5
 d. dne
 e. $\frac{1}{2}$
 f. $\frac{1}{2}$
 g. $\frac{1}{2}$
 h. 0
 i. dne
 j. $(-4, -1) \cup (-1, 1) \cup (1, 2) \cup (2, 4)$