**7 - Continuity**

**show your work in your notebook!!**

Find the limit if it exists.

1. $\lim\limits_{\Delta x \to 0^-} \dfrac{\dfrac{1}{x+\Delta x} - \dfrac{1}{x}}{\Delta x}$

2. $\lim\limits_{x \to 3^-} f(x)$ where $f(x) = \begin{cases} \dfrac{x+2}{2}, & x \le 3 \\ \dfrac{12-2x}{3}, & x > 3 \end{cases}$

3. $\lim\limits_{x \to 2} f(x)$ where $f(x) = \begin{cases} x^2 - 4x + 6, & x < 2 \\ -x^2 + 4x - 2, & x \ge 2 \end{cases}$

Find a that would make the function continuous.

4. $f(x) = \begin{cases} x^3, & x \le 2 \\ ax^2, & x > 2 \end{cases}$

5. $f(x) = \begin{cases} 2, & x \le -1 \\ ax + b, & -1 < x < 3 \\ -2, & x > 3 \end{cases}$

Use the Intermediate Value Theorem to determine if there is a zero on the given interval. Find the zero using your graphing calculator, if it exists.

6. $f(x) = x^3 + x - 1$, $[0,1]$
7. $f(x) = x^3 + 3x - 2$, $[0,1]$

Verify that the Intermediate Value Theorem applies on the given interval and find the value of c guaranteed by the theorem.

8. $f(x) = x^2 + x - 1$, $[0,5]$, $f(c) = 11$
9. $f(x) = x^2 - 6x + 8$, $[0,3]$, $f(c) = 0$
10. $f(x) = x^3 - x^2 + x - 2$, $[0,3]$, $f(c) = 4$

Name any vertical asymptotes.

11. $f(x) = \dfrac{4}{(x-2)^2}$

12. $f(x) = \dfrac{2+x}{x^2(1-x)}$

13. $f(x) = \dfrac{-4x}{x^2+4}$

14. $g(t) = \dfrac{t-1}{t^2-1}$

15. $f(x) = \sin \pi x$ (Hint: Graph it!)

16. $f(x) = \dfrac{4x^2 + 4x - 24}{x^4 - 2x^3 - 9x^2 + 18x}$

Find the limit.

17. $\lim\limits_{x \to 2^+} \dfrac{x-3}{x-2}$

18. $\lim\limits_{x \to 3^+} \dfrac{x^2}{x^2-9}$

19. $\lim\limits_{x \to -3^-} \dfrac{x^2+2x-3}{x^2+x-6}$

20. $\lim\limits_{x \to 1} \dfrac{x^2 - x}{(x^2+1)(x-1)}$

21. $\lim\limits_{x \to 0^-} \left(1 + \dfrac{1}{x}\right)$

22. $\lim\limits_{x \to 0^+} \dfrac{2}{\sin x}$

**ANSWERS**

1. $\dfrac{-1}{x^2}$
2. 5/2
3. 2
4. $a = 2$
5. $a = -1,\ b = 1$
6. $f(0) = -1,\ f(1) = 1,\ x \approx .6823$
7. $f(0) = -2,\ f(1) = 2,\ x \approx .5961$
8. $c = 3$
9. $c = 2$
10. $c = 2$
11. $x = 2$
12. $x = 0,1$
13. None
14. $-1$ only
15. Sine has no asymptotes!
16. $x = 0,3$
17. $-\infty$
18. $\infty$
19. 4/5
20. ½
21. $-\infty$
22. $\infty$