6 - Limits and Continuity

The 3-Step Continuity Test

1. $f(x)=\begin{cases} x^2+4 \ ; \ x\ge 1 \\ 3x+2 \ ; \ x<1 \end{cases}$
   a. Sketch a graph $y=f(x)$.
   b. $f(1)$ = ?
   c. $\lim\limits_{x\to 1^-} f(x)=$
   d. $\lim\limits_{x\to 1^+} f(x)=$
   e. Is f continuous at $x=1$? Give a reason to justify your answer.

2. $g(x)=\begin{cases} x^2-2 \ ; \ x\le 2 \\ 1-x \ ; \ x>2 \end{cases}$
   a. Sketch a graph $y=g(x)$.
   b. Is f continuous at $x=2$? Give a reason to justify your answer.

Perform the 3 step test for continuity at a point for the following functions:

3. $f(x)=\begin{cases} 3x^2+4x+5; x>1 \\ x+11; x<1 \end{cases}$

4. $g(x)=\begin{cases} 4x-2; x\le 3 \\ 3-x^2; x>3 \end{cases}$

5. $h(x)=\begin{cases} 2-x; x<-2 \\ 0; x=-2 \\ x^3-3x+6; x>-2 \end{cases}$

6. $i(x)=\begin{cases} x^2-2x+2; x>3 \\ 5; x=3 \\ 5x-10; x<3 \end{cases}$

7-12 the value of a,b or c to make the functions continuous.

7. $h(x)=\begin{cases} cx^2-3 \ ; \ x\le 2 \\ cx+2 \ ; \ x>2 \end{cases}$

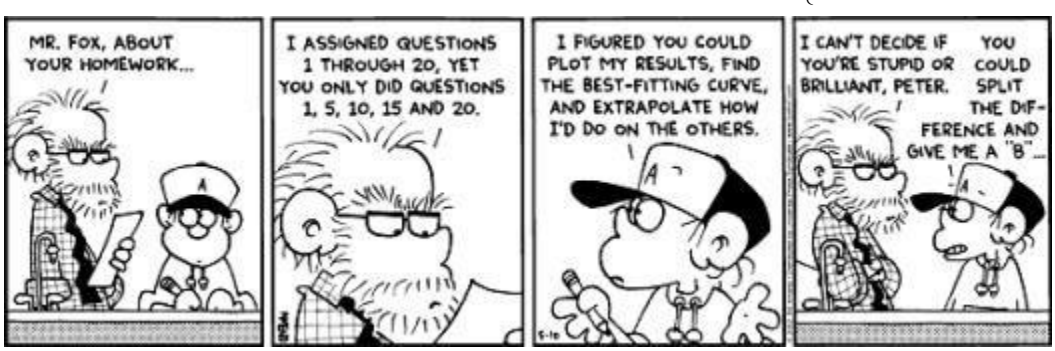
8. $i(x)=\begin{cases} ax+b \ ; \ x>3 \\ 2 \ ; \ x=3 \\ 2bx-a \ ; \ x<3 \end{cases}$

9. $f(x)=\begin{cases} x^3; x\le 2 \\ ax^2; x>2 \end{cases}$

10. $g(x)=\begin{cases} 2; x\le -1 \\ ax+b; -1<x<3 \\ -2; x\ge 3 \end{cases}$

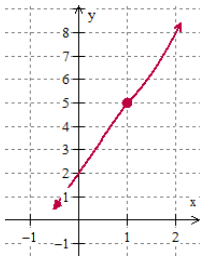
11. $h(x)=\begin{cases} \dfrac{4\sin x}{x}; x<0 \\ a-2x; x\ge 0 \end{cases}$

12. $i(x)=\begin{cases} \dfrac{x^2-a^2}{x-a}; x\ne a \\ 8; x=a \end{cases}$



ANSWERS

1. –

   a. 

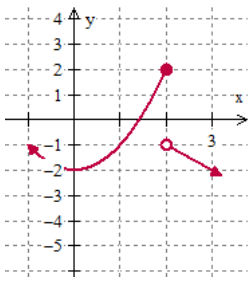
   b. $f(1)=5$

   c. 5

   d. 5

   e. Continuous since $f(1)=5=\lim_{x\to 1} f(x)$.

2. –

   a. 

   b. $\lim_{x\to 2^-} f(x)=-1$, $\lim_{x\to 2^+} f(x)=2$

   So, f is not continuous at 2.

3. –

   1. $f(1)$ does not exist.
   2.
   3.

   Conclusion: $f(x)$ is not continuous at $x=1$

4. 1. $g(1)=10$

   2. $\lim_{x\to 3^+} g(x)=0$, $\lim_{x\to 3^-} g(x)=10$

   3.

   Conclusion: $g(x)$ is not continuous at $x=3$.

5. 1. $h(-2)=0$

   2. $\lim_{x\to -2^+} h(x)=4$, $\lim_{x\to -2^-} h(x)=4$

   3. $h(-2)=0\neq \lim_{x\to -2} h(x)=4$

   Conclusion: $h(x)$ is not continuous at $x=-2$.

6. 1. $i(3)=5$

   2. $\lim_{x\to 3^+} i(x)=5$, $\lim_{x\to 3^-} i(x)=5$

   3. $i(3)=5=\lim_{x\to 3} i(x)$

   Conclusion: $i(x)$ is continuous at $x=3$.

7. $c=\frac{5}{2}$

8. $a=\frac{4}{5}, b=\frac{2}{5}$

9. $a=2$

10. $a=-1, b=1$

11. $a=4$

12. $a=4$