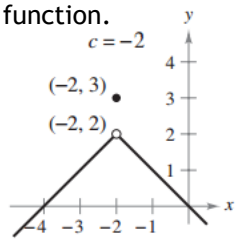
5 - Continuity and One Sided Limits

Determine the limits and discuss the continuity of the function.

1.

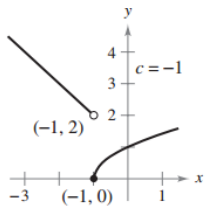


a. $\lim_{x\to c^+} f(x) =$

b. $\lim_{x\to c^-} f(x) =$

c. $\lim_{x\to c} f(x) =$

2. -



a. $\lim_{x\to c^+} f(x) =$

b. $\lim_{x\to c^-} f(x) =$

c. $\lim_{x\to c} f(x) =$

Find the limit if it exists.

3. $\lim_{x\to 5^+} \dfrac{x-5}{x^2-25}$

4. $\lim_{x\to 2^+} \dfrac{2-x}{x^2-4}$

5. $\lim_{x\to 0^+} \dfrac{|x|}{x}$

6. $\lim_{x\to 4^-} \dfrac{\sqrt{x}-2}{x-4}$

7. $\lim_{x\to 2^-} \dfrac{|x-2|}{x-2}$

8. $\lim_{\Delta x\to 0^-} \dfrac{\dfrac{1}{x+\Delta x}-\dfrac{1}{x}}{\Delta x}$

9. $\lim_{\Delta x\to 0^+} \dfrac{(x+\Delta x)^2+x+\Delta x-(x^2+x)}{\Delta x}$

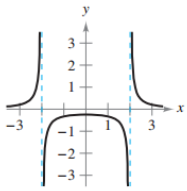
10. $\lim_{x\to 3^-} f(x)$, where $f(x)=\begin{cases} \dfrac{x+2}{2}, & x\le 3 \\ \dfrac{12-2x}{3}, & x>3 \end{cases}$

11. $\lim_{x\to 2} f(x)$, where $f(x)=\begin{cases} x^2-4x+6, & x<2 \\ -x^2+4x-2, & x\ge 2 \end{cases}$

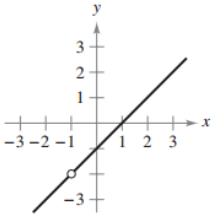
12. $\lim_{x\to 1^+} f(x)$, where $f(x)=\begin{cases} x^3-7, & x<1 \\ x+1, & x\ge 1 \end{cases}$

Discuss the continuity of each function.

13. $f(x) = \dfrac{1}{x^2-4}$



14. - $f(x) = \dfrac{x^2-1}{x+1}$



Find x-values where each is NOT continuous. Label each as removable or non-removable.

15. $f(x)=\dfrac{1}{x^2+1}$

16. $f(x)=\cos\dfrac{\pi x}{2}$

17. $f(x)=\dfrac{x}{x^2-1}$

18. $f(x)=\dfrac{x-3}{x^2-9}$

19. $f(x)=\dfrac{|x+2|}{x+2}$

20. $f(x)=\begin{cases} x; & x\le 1 \\ x^2; & x>1 \end{cases}$

21. $f(x)=\begin{cases} -2x+3; & x<-1 \\ x^2; & x\ge -1 \end{cases}$

22. $f(x)=\begin{cases} \tan\dfrac{\pi}{4}x; & |x|<1 \\ x; & |x|>1 \end{cases}$

Answers

1. –
   a. 2
   b. 2
   c. 2
2. –
   a. 0
   b. 2
   c. Dne
3. 1/10
4. -1/4
5. 1
6. ¼
7. -1
8. $-\frac{1}{x^2}$
9. 2x+1
10. 5/2
11. 2
12. 2
13. Non removable discontinuity at x = 2,-2
14. Removable discontinuity at x = 1
15. No discontinuity
16. No discontinuity
17. Non-removable discontinuity at x = 1,-1
18. Removable at x = 3, Nonremovable discontinuity at x = -3
19. Non removable at x = -2
20. No discontinuity
21. Non removable at x = -1
22. Removable at x = 1, nonremovable discontinuity at x = -1