LO 3.2A Interpret the definite integral as the limit of a Riemann sum. Express the limit of a Riemann sum in integral notation.

Instructions: Match the integral expression in the left column with the appropriate limit of a Riemann sum in the right column.

1. $\int_1^3 (4x^2 + 2)\,dx$	_____	**a.** $\lim_{n\to\infty} \sum_{j=1}^{n} \left[4\left(5 - \frac{3j}{n}\right) - 2\right]\left(\frac{-3}{n}\right)$
2. $\int_2^5 (x^3 + 1)\,dx$	_____	**b.** $\lim_{n\to\infty} \sum_{j=1}^{n} \left[4\left(2 + \frac{3j}{n}\right)^2 + 2\right]\left(\frac{3}{n}\right)$
3. $\int_7^5 (3x + 1)\,dx$	_____	**c.** $\lim_{n\to\infty} \sum_{j=1}^{n} \left[\left(2 + \frac{3j}{n}\right)^3 + 1\right]\left(\frac{3}{n}\right)$
4. $\int_2^4 (4x^2 + 2)\,dx$	_____	**d.** $\lim_{n\to\infty} \sum_{j=1}^{n} \left[4\left(1 + \frac{2j}{n}\right)^2 + 2\right]\left(\frac{2}{n}\right)$
5. $\int_5^2 (4x - 2)\,dx$	_____	**e.** $\lim_{n\to\infty} \sum_{j=1}^{n} \left[3\left(7 - \frac{2j}{n}\right) + 1\right]\left(\frac{-2}{n}\right)$
6. $\int_2^5 (4x^2 + 2)\,dx$	_____	**f.** $\lim_{n\to\infty} \sum_{j=1}^{n} \left[4\left(3 - \frac{2j}{n}\right)^2 + 2\right]\left(\frac{-2}{n}\right)$
7. $\int_5^7 (4x - 2)\,dx$	_____	**g.** $\lim_{n\to\infty} \sum_{j=1}^{n} \left[3\left(2 + \frac{3j}{n}\right) + 1\right]\left(\frac{3}{n}\right)$
8. $\int_3^1 (4x^2 + 2)\,dx$	_____	**h.** $\lim_{n\to\infty} \sum_{j=1}^{n} \left[4\left(5 + \frac{2j}{n}\right) - 2\right]\left(\frac{2}{n}\right)$
9. $\int_5^7 (x^3 + 1)\,dx$	_____	**i.** $\lim_{n\to\infty} \sum_{j=1}^{n} \left[\left(5 + \frac{2j}{n}\right)^3 + 1\right]\left(\frac{2}{n}\right)$
10. $\int_2^5 (3x + 1)\,dx$	_____	**j.** $\lim_{n\to\infty} \sum_{j=1}^{n} \left[4\left(2 + \frac{2j}{n}\right)^2 + 2\right]\left(\frac{2}{n}\right)$

4 - AREAS AND VOLUME BY SLICES

Find the enclosed area without a calculator. Draw a graph of the region.

1. $y=2x^2$ and $y=2x$
2. $y=x^2-2x+4$ and $y=x+4$
3. $y=x^3-3x^2-2x+1$ and $y=2x+1$

Find the area with a calculator. Draw a labeled graph of the region.

4. $h(x)=2x^2-3x-3$ and $j(x)=-\frac{1}{3}x^2-2x+2$
5. $f(x)=\sin(0.5x)-0.2x-1$ and $g(x)=x^3-4x-1$
6. $u(x)=x^4-3x^3-2x^2+6x+2$ and $m(x)=2\sin\left(0.25\left(x^2-1\right)\right)+2$

Find the volume with the given slices without a calculator.

7. Find the volume of the solid that is bounded by the circle $x^2+y^2=9$ with the indicated cross sections taken perpendicular to the x-axis. Draw a 3 dimensional picture of a and c.
    a. Squares
    b. Equilateral triangles
    c. Semicircles
    d. Isosceles right triangles

8. Find the volume of the solid that is bounded by $y=x^2$, $y=0$ and $x=2$ with the indicated cross sections taken perpendicular to the x-axis.
    a. Squares
    b. Rectangles whose height is twice their base
    c. Semicircles
    d. Rectangles of height 2

Find the volume with the given slices **WITH A CALCULATOR**. Round to 3 decimal places.

9. Base the area ENCLOSED by $g(x)=2\sin(1.8x)-1$ and $f(x)=-0.36x^2-0.15x+2.8$
    a. Squares
    b. Rectangles of height 3
    c. Semicircles
    d. Right isosceles triangles
    e. Equilateral triangles
    f. Rectangles 1/2 as tall as wide

10. Base enclosed by the x and y axis and $f(x)=4-x^2$
    a. Squares
    b. Rectangles of height 3
    c. Semicircles
    d. Right isosceles triangles
    e. Equilateral triangles

ANSWERS

SIDE2

1. d
2. c
3. e
4. j
5. a
6. b
7. h
8. f
9. i
10. g

SIDE1

1. 1/3
2. 9/2
3. 131/4
4. 10.074
5. 9.063
6. 13.905
7. –
    a. 144
    b. $36\sqrt{3}$
    c. 18π
    d. 72
8. –
    a. 32/5
    b. 64/5
    c. $\frac{4\pi}{5}$
    d. 16/3
9. –
    a. 62.139
    b. 50.357
    c. 24.402
    d. 31.070
    e. 26.907
    f. 31.070
10. –
    a. 17.067
    b. 16
    c. 6.702
    d. 8.533
    e. 7.390