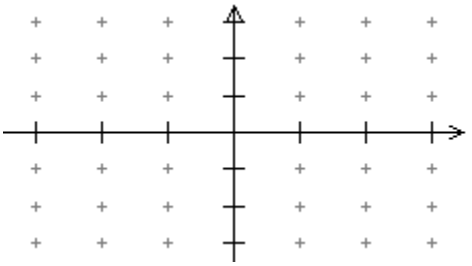
WS SLOPE FIELDS, DIFFERENTIAL EQUATIONS, INVERSE TRIG FUNCTIONS

1. Consider the differential equation $\frac{dy}{dx}=\frac{xy}{2}$.



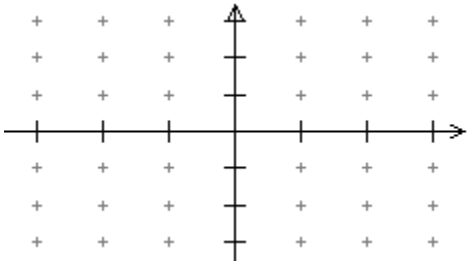
   a. On the axis at the right, sketch a slope field.

   b. Let f be the function that satisfies the differential equation. Write an equation for the tangent line to the curve $y=f(x)$ through the point (1,1). Then use your tangent line to estimate the value of $f(1.2)$ (plug in 1.2 for x in the tangent line equation).

   c. Find the particular solution to $y=f(x)$ to the differential equation with initial condition $f(1)=1$. Use the solution to find $f(1.2)$.

   d. Compare your estimate in part b to the actual answer in part c. Is it an over or under estimate? Use your slope field to answer why.

2. Consider the differential equation $\frac{dy}{dx}=-2yx$.



   a. On the axis at the right, sketch a slope field.

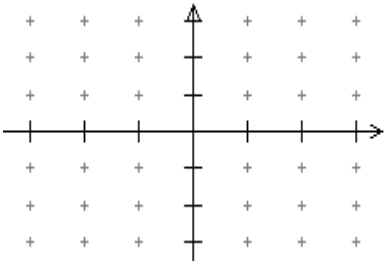
   b. Let f be the function that satisfies the differential equation. Write an equation for the tangent line to the curve $y=f(x)$ through the point (1,1). Then use your tangent line to estimate the value of $f(1.2)$.

   c. Find the particular solution to $y=f(x)$ to the differential equation with initial condition $f(1)=1$. Use the solution to find $f(1.2)$.

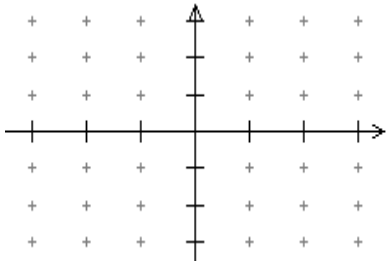
3. Compare your estimate in part b to the actual answer in part c. Is it an over or under estimate? Use your slope field to answer why.

Draw the slope field from its equation. Draw that particular solution that passes through (1,1).

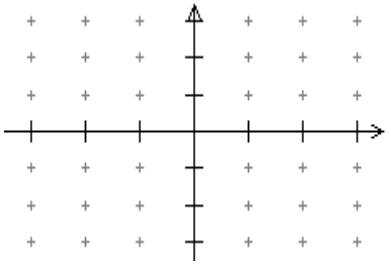
a. $\frac{dy}{dx}=\frac{-2}{x}$



b. $\frac{dy}{dx}=1+x+y$



c. $\frac{dy}{dx}=e^x$



Solve the differential equation.

4. $y' = x - 2$

5. $\left(1 + x^3\right)y' - 3x^2y = 0$

6. $y' = xy$

7. $y' = \frac{y}{x}$

8. $y' = \frac{2x}{y}$

9. $y' = y\ \sqrt[3]{x}$

10. $y' = 2x\left(1 + y\right)$

11. $xy + y' = 2x$

12. $\left(1 + x^3\right)y' - 3x^2y = 0$

13. $\frac{dy}{dt} = ykt$

Solve the differential equation if it passes through the point (0,2).

14. $\frac{dy}{dt} = \frac{1}{3}y$

15. $\frac{dy}{dt} = \frac{1}{3}t$

Answers

1. –
   a. –
   b. $y-1=\frac{1}{2}(x-1)$, $f(1.2)\approx 1.1$
   c. $y=\frac{1}{e^{\frac{1}{4}}}e^{\frac{1}{4}x^2}$, $f(1.2)=1.116$
   d. Underestimate; at (1,1), f is concave up
2. –
   a. –
   b. $y-1=-2(x-1)$, $f(1.2)\approx .6$
   c. $y=e\cdot e^{-x^2}$, $f(1.2)=.644$
   d. Underestimate; at (1,1), f is concave up
3. https://www.geogebra.org/m/W7dAdgqc
4. $y=\frac{1}{2}x^2-2x+C$
5. $y=C\left|1+x^3\right|$
6. $y=Ce^{\frac{1}{2}x^2}$
7. $y=C|x|$
8. $y^2=2x^2+C$
9. $y=Ce^{\frac{3}{4}x^{\frac{4}{3}}}$
10. $y=Ce^{x^2}-1$
11. $y=2-Ce^{-\frac{1}{2}x^2}$
12. $y=C\left|1+x^3\right|$
13. $y=Ce^{\frac{1}{2}kt^2}$
14. $y=2e^{\frac{1}{3}x}$
15. $y=\frac{1}{6}x^2+2$